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Study on Transverse Horizontal Error Calculation and Alignment Fitting for Newly Built Railway

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Abstract
The design alignment of newly built railway is described by sorted linked lists consisting of straight line element, circular curve element and transition curve element. Based on this description this article studies the method for calculating transverse horizontal errors of structure centers of newly built railway. The article also studies the method for fitting newly built railway alignment, that is based on fitting straight line element and circular curve element using the least squares method and then resolving the transition curve element.

Key words: Transverse error; Least squares method; Alignment fitting

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INTRODUCTION

In the reconnaissance survey of newly built railway lines such as Wuhan-Guangzhou high speed railway and Zhengzhou-Xi'an high speed railway, in accordance with the survey codes then in effect, after adjustment the tolerance of relative misclosure was 1:20000. In October 2006, a provisional regulation coded TJS 189 (2006) was enacted by the Ministry of Railway of China, which mandates the reestablishment of a precise survey network. The provisional regulation stipulates a new tolerance of total relative misclosure of 1:40000 for the route control point network (CPII), which is used for the purpose of route reconnaissance survey and horizontal control in the construction stage. By the time the new regulation was enacted, construction had already commenced for structures like bridge piers and abutments. To meet the new requirement of horizontal control precision, a new alignment must be fitted to ensure a relatively small transverse horizontal error of the structure centers, and to provide the adjusting direction and value for structures.

In search of a solution to this challenge, this article aims to study the method for calculating transverse horizontal error of newly built railway structure centers, as well as the alignment fitting technique.

1. DESCRIPTION OF DESIGN LINE

The design line of the newly built railway consists of straight line element, circular curve element and transition curve element that are formed by connecting straight line, transition curve and circular curve in the proper order. These are basic elements of the horizontal alignment of newly built railways and in this article are referred to as "line elements", described by the following parameters:

Straight Line Element (SLE): continuous mileage at starting point, starting point coordinates, azimuth, length;

Circular Curve Element (CCE): continuous mileage at starting point, starting point coordinates, starting tangent azimuth, left/right turning, radius, length;

transition curve element (TCE): continuous mileage at starting point, starting point coordinates, starting tangent azimuth, left/right turning, starting point radius, end point radius, length;

A design line can be described by a sorted linked lists consisting of line elements.

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2. METHOD FOR CALCULATING TRANSVERSE HORIZONTAL ERROR OF STRUCTURE CENTER

Transverse horizontal error of structure center refers to the projection distance from the structure center to the railway line center. In this article, when the structure center deviates towards the left side of the line, the error is marked as minus; otherwise it is marked as plus. The calculation of transverse error is done in two steps:

(a) Locate the line element to which the structure center corresponds, i.e. determine on which line element the structure center's projection point falls.

(b) Calculate the positional relationship between the structure center and the line element concerned, including the projection distance from the structure center to the line element, and whether the structure center is located on the left side or the right side of the route.

2.1 Locate the Structure Center's Corresponding Line Elements

Traverse all line elements in the proper order and assume there are two points along the normal at the starting point of a line element: A (X_1, Y_1) and B (X_2, Y_2). Point A is located on the left side of the route with respect to its advancing direction and point B is located on the right side of the route with respect to its advancing direction. Assume there are two points along the normal at the end point of the line element: C (X_3, Y_3) and D (X_4, Y_4). Point C is located on the left side of the route with respect to its advancing direction and point D is located on the right side of the route with respect to its advancing direction. The coordinates of structure center P is (X_0, Y_0) (see Figure 1).

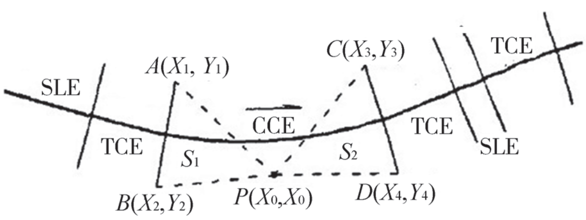

Figure 1
Structure Center's Corresponding Line Elements

Calculate ΔPAB:

$S_1=0.5\times[(Y_2-Y_1)\,X_0-(X_2-X_1)\,Y_0-X_1Y_2+X_2Y_1]$.

Calculate ΔPCD:

$S_2=0.5\times[(Y_4-Y_3)\,X_0-(X_4-X_3)\,Y_0-X_3Y_4+X_4Y_3]$.

Based on the values of S_1 and S_2 determine if the structure center (P)'s projection point on the route falls on the line element:

(a) If S_1=0, the projection point of P on the route coincides with the starting point of the line element.

(b) If S_2=0, the projection point of P on the route coincides with the end point of the line element.

(c) If S_1<0 and S_2>0, the projection point of P on the route falls on the line element.

(d) If S_1>0 or S_2<0, the projection point of P on the route does not fall on the line element.

2.2 Analysis of Relationship Between Structure Center and Line Element

2.2.1 Relationship Between Point and Straight Line Element

Assume the starting point of a straight line element is A (X_1, Y_1), the end point is B (X_2, Y_2), and the structure center's coordinates are P (X_0, Y_0), as shown in Figure 2. The distance between P and the straight line AB can be calculated by the following formula:

Figure 2
Relationship Between Point and Straight Line Element

$$d=\frac{(Y_2-Y_1)X_0-(X_2-X_1)Y_0-X_1Y_2+X_2Y_1}{\sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2}}$$

When d>0, P is located on the left side of AB; when d<0, P is located on the right side of AB.

2.2.2 Relationship Between Point and Circular Curve Element

Assume the starting point of a circular curve is A (X_1, Y_1), starting point tangent azimuth is f, left turning/right turning is k (k=-1 for turning left, k=+1 for turning right), radius is R, and length is L_c; the structure center's coordinates are P (X_0, Y_0), as shown in Figure 3. Based on their Euclidean geometry relationship, the following evaluations can be done:

Coordinates of center of circle O:

$$\begin{cases} Xc=X_1+R\cdot\sin(f+k\cdot\pi/2)\,, \\ Yc=X_1+R\cdot\cos(f+k\cdot\pi/2)\,. \end{cases}$$

Distance between O and P:

$$D_{op}=\sqrt{(X_o-X_c)^2+(Y_o-Y_c)^2}.$$

Since the structure center is located near the route, the distance between P and the circular curve element is D=.

If $k\cdot(R\text{-}D_{op})$>0, P is located on the right side of the route center line.

If $k\cdot(R\text{-}D_{op})$<0, P is located on the left side of the route center line.

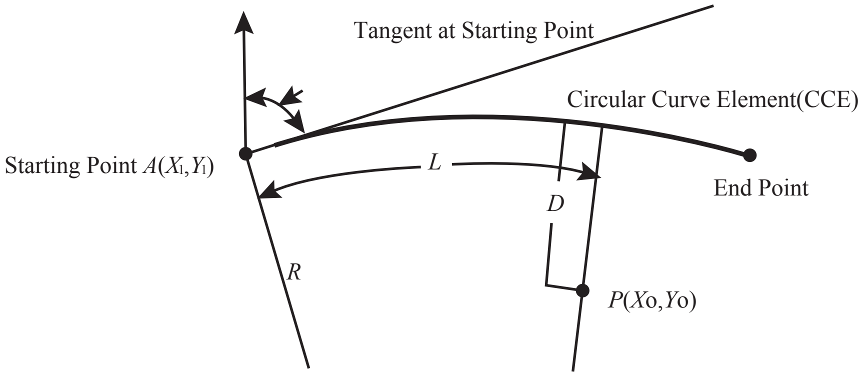

Figure 3
Relationship Between Point and Circular Curve Element

2.2.3 Relationship Between Point and Transition Curve Element

The transition curve of the newly built railway is a radial spiral (referred to as clothoid in the road sector). The basic characteristics of the radial spiral are that, at any point on the transition curve, product of the curvature radius ρ, and the arc length *l* between the point concerned and the transition curve's starting point, is a constant, i.e. $\rho\cdot l=A^2$ where A is referred to as the clothoid parameter. The formula to calculate the coordinates of any point on the transition curve can be deduced (Zhan, 2001) from Figure 4 as:

$$x=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}1^{4n-3}}{(2n-2)!2^{2n-2}A^{4n-4}(4n-3)},$$

$$y=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}1^{4n-3}}{(2n-2)!2^{2n-2}A^{4n-2}(4n-1)}. \quad (1)$$

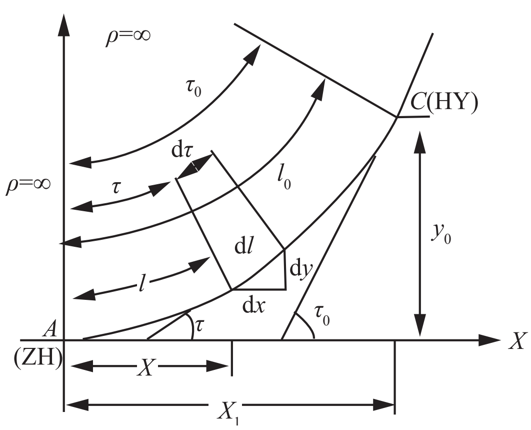

Figure 4
Characteristics of Transition Curve

The tangent rotation r at any point on the transition curve with respect to the starting point (transition point):

$$r=0.5\cdot l^2/A^2, \quad (2)$$

In the case of transition curve element, when the starting point coordinates, starting point tangent azimuth, left/right turning, starting point radius, end point radius, and length are given, the coordinates of any point on the transition curve element can be calculated by Formula (1). The tangent azimuth of any point on the transition curve element can be calculated by Formula (2). The relationship between the structure center and the transition curve element can thereby be solved using the "bisection method" following these steps:

Step 1

Assume the length of transition curve is *length*, set the initial search scope as [*From*, *To*] where *From*=0 (starting point of transition curve element) and *To*=*length* (end point of transition curve element).

Step 2

If $|From,-T_O|<0.00001$, move to Step 3; otherwise calculate the coordinates and tangent azimuth of M_i, the midpoint between *From* and *To*. Then calculate two points on the normal of M_i: L_i (X_1, Y_1) on the left side of the route and R_i (X_2, Y_2) on the right side of the route. Assume the coordinates of the structure center *P* are (X_0, Y_0), and the area of ΔPL_iR_i is calculated as:

$$S_i=0.5\times[(Y_2-Y_1)X_0-(X_2-X_1)X_0-X_1X_2+X_2X_1].$$

If $S_i>0$, *P* is located on the left side of straight line R_iL_i, *To*=(*From*, *To*)/2, move to Step 2;

If $S_i<0$, *P* is located on the right side of straight line R_iL_i, *To*=(*From*, *To*)/2, move to Step 2.

Step 3

According to 2.2.1 determine the relationship between *P* and the straight line segment between *From* and *To,* which can be regarded as the relationship between *P* and the transition curve element.

3. ALIGNMENT FITTING METHOD

The fitting of straight line element and circular curve element can be done using the least squares method, which means the sum of the squares of errors of the fitted straight line or curve and the sample data is minimized (The Editorial Committee of Modern Applied para.2, 2005, pp.119-128). The transition curve element can be solved according to the straight line element and the circular curve element.

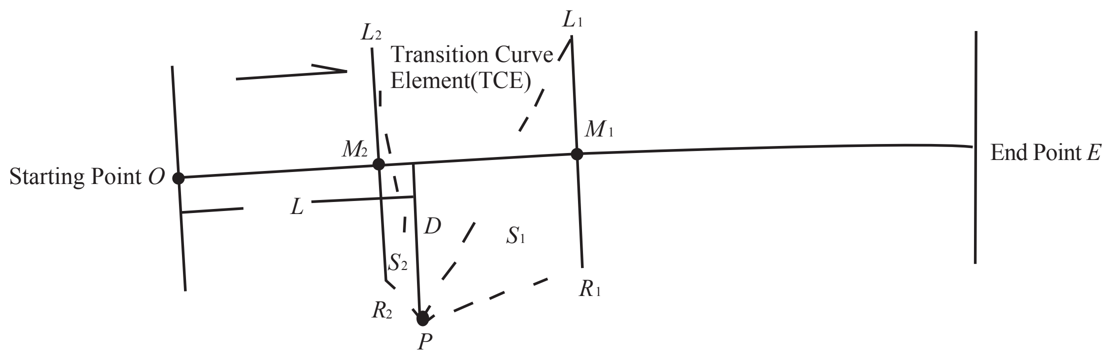

Figure 5
Calculating the Relationship Between Point and Transition Curve Element Using Bisection Method

3.1 Fitting of Straight Line using Least Squares Method

Sample (X_1, Y_1), (X_2, Y_2) … (X_i, Y_i) … (X_n, Y_n). Assume the straight line equation is $aX+b=Y$. According to the least squares method, a system of linear equations in a and b can be established:

$$\begin{cases} a\sum_{i=1}^{n} Xi^2 + b\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} YiXi \\ a\sum_{i=1}^{n} Xi + nb = \sum_{i=1}^{n} Yi \end{cases} . \quad (2)$$

Solve a and b of the above equations, and then the equation of the target straight line can be established.

When the angle K (formed by the straight line and Axis X), 90°, the fitting error is relatively large. Therefore a prediction of the angle K should be made prior to the fitting. If $45°<K<135°$, reverse coordinate X and Y prior to the fitting. Rotate the fitted straight line by 180° about the origin and the result will be the target straight line.

3.2 Fitting of Circle by Least Squares Method

Sample (X_1, Y_1), (X_2, Y_2) … (X_i, Y_i) … (X_n, Y_n). Assume the circular curve equation is $(X-A)^2+(Y-B)^2=R^2$, let $a=-2A$, $b=-2B$, $c=A^2+B^2-R^2$ and a system of linear equations in a, b and c can be established (The Editorial Committee of Modern, para.3, 2005, pp.119-128):

$$\begin{cases} \sum_{i=1}^{n}(X_i^2 + Y_i^2 + aX_i + bY_i + c)X_i = 0 \\ \sum_{i=1}^{n}(X_i^2 + Y_i^2 + aX_i + bY_i + c)Y_i = 0 \\ \sum_{i=1}^{n}(X_i^2 + Y_i^2 + aX_i + bY_i + c) = 0 \end{cases} . \quad (4)$$

Solve a, b, and c of the above equations and then solve A, B, and R; then the equation of target circular curve can be established.

3.3 Solving Transition Curve

As shown in Figure 6, when the positions of the straight line element and circular curve element are determined, i.e. the coordinates of two points (X_1, Y_1), (X_2, Y_2) on the straight line element, the coordinates of the center of circle (X_c, Y_c) of the circular curve element, and the radius R are given, the value A of the transition curve element connecting the straight line element and the circular curve element can be solved as follows:

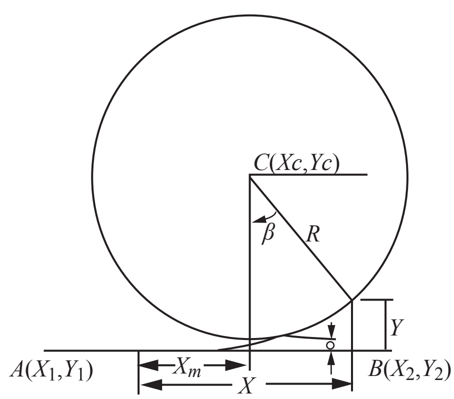

Figure 6
Straight Line and Circular Curve Connected by Transition Curve

(a) Solve the distance from the center of circle to the straight line:

$$D = \left| \frac{k(X_C - X_1) - (Y_C - Y_1)}{\sqrt{1 + k^2}} \right| ,$$

$$k = \frac{Y^2 - Y_1}{X_2 - X_1} . \quad (5)$$

When $X_2=X_1$, $D=|X_C-X_1|$.

(b) Establish the mathematical relationship between A (curve element value) and D (distance between the center of circle C and the straight line) (Miu & Zhan, 2012; Yang, 2004):

$$D = Y + R\cos\left(\frac{A^2}{2R^2}\right), \quad (6)$$

which can be further converted to:

这幅图很不清楚，请重新提供清晰的

$$D = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} A^{4n}}{(2n-1)!\,2^{n-1} R^{4n-1}(4n-1)} + R\cos\left(\frac{A^2}{2R^2}\right).$$

The mathematical implication of the above formula is that, when the circular curve radius is given, for any value of A (A') of the transition curve element, there is a D' (distance between the center of circle C and the straight line) corresponding to it. Figure 7 shows the change of D dependent on A.

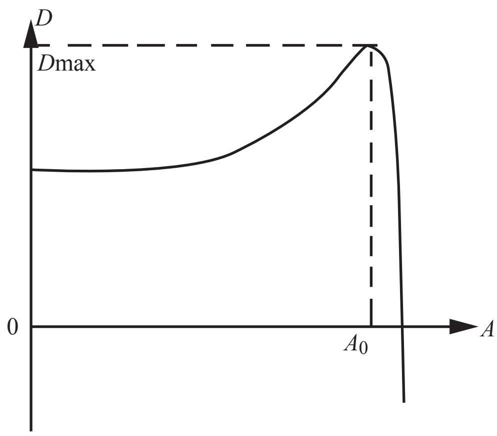

Figure 7
Relationship Between Circle Center - Straight Line Distance and Value *A*

(c) Solve the reasonable value A of the transition curve element using the numerical solution method.

First, solve the maximum point (A_0, D_{max}) using the golden section search method, then solve the point (A, D) of the interval using the bisection method, and value A of the transition curve can be solved which meets the requirement that the distance between the straight line and the center of circle is D) (Peng, 2004).

CONCLUSION

Based on this article's study on the method for calculating transverse horizontal error of structure centers, and the alignment fitting technique for newly built railway, a computer software was developed (Figure 8) and has found wide application by the China Railway Siyuan Survey and Design Group Co., Ltd in projects such as the Wuhan-Guangzhou high speed railway and the Zhengzhou-Xi'an high speed railway.

Figure 8
Software Screenshot

The method and technique described by this article have been proved in practice to be correct and efficient, and able to save designers copious amount of time on trial calculation, hence reducing design cycles. The alignment fitted is able to ensure a relatively small transverse horizontal error of the structure center, and to provide guidance to structure adjustment. The method and software can also be used in the design of railway reconstruction.

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