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Analysis and Application of Extension Correlation and Correspondence under Uncertainty

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Abstract: This paper studies the meaning, nature of extension correspondence and analyzes its superiority in data prediction in extension group decision-making by analyzing the distance and bit value of the correlation function. It puts forward the model and steps of data interval analysis of extension group decision-making based on extension correspondence. Not only does it take the advantages of dynamic classification in extension group decision-making, but also realizes data analysis and interval estimation under uncertainty and helps the promotion of the accuracy and the reliability of multi-factor analysis and multi-project evaluation in extension group decision-making under data uncertainty.
Keywords: extension set; extension group decision-making; correlation function; correspondence analysis; uncertainty

1. INTRODUCTION

Extenics is a new science subject (CAI Wen, Sep.1999), research on the extension possibility and extension laws of things to resolve contradictions problems. Its research object is the contradictions problems in real world and theoretical base is matter-element and extension set theory.

In classical mathematics, with the characteristic function to describe whether the elements in domains having a certain nature or not; in fuzzy mathematics, with the membership function to characterize it having the extent of a certain nature; in extenics, with the correlation function to portray it so. Correlation function indicates that when the value of matter-element is a point in solid axis, with matter-element to meet the requirements of the value range, it can quantitatively and objectively describe the elements having the nature or character at a certain extent and the process of quantitative change and qualitative change. (WANG Ze-min, Jun.1998) succeeded in promoting the concept of extension set to n-dimensional Euclidean space,and analyzed the nature and application of the n-variables correlation

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function. [3] studied the concept of correlation function space $\Omega(\overline{X})$ and discussed the properties of $\Omega(\overline{X})$, obtained the product theorem and combination theorem of the $\lambda(x)$. (HU, WANG, HE, June 2000) introduced concepts of extension set, interval distance and interval place distance on intervals and discusses their properties. The extension set based on correlation function has successfully applied to the forecast of container accumulation (WANG Chao, No.10, 2008), classifying of reservoir flow unit (LI, WANG, Dec.2007), cluster evaluation for ore rock cavability (DENG, ZHOU, Jan. 2008), product improvement design (HUANG, LIN, Jan.2008), risk investment decision (BAI, Jun.2008), knowledge acquisition (YANG, CAI, Aug.2008), data mining (CHEN, 2006), operational command (ZHANG Su, 2007), multiple fault diagnosis (JIN, CHEN, JIN, August 2006), etc.

Extension group decision-making, a problem of group decision-making described with matter-element extension set, studies extension associated and extension decision-making space, compatibility and conflict changes among decision-making systems by combining extension transformation with group decision optimization. Thus it produces the relevant policy strategies and realizes comparison and selection of objects in changing environment. Correlation function is an important component of extension group decision-making, through which studies the correspondence of extension associated. Then it can lead to the data interval analysis under uncertainty, by which can not only estimate the possible range of missing data based on available data, but also predict the efficient space of missing data under a certain constraint condition.

2. CORRELATION FUNCTION

Through the correlation function can quantitatively describe the area to which any one element belongs in the domain. For elements belonging to a same region distinguish from different levels by values size of the correlation function, as is of great significance to research the properties and changes of the elements under uncertainty.

2.1 Distance and position

Definition 1: if $|X_1| = |b-a|$ is the modulus of bounded interval $X_1 = (a,b)$, then $|x| = 0$ is the modulus of point x.

Nature 1: if the interval $X_1 \supseteq X_2$, then $|X_1| \geq |X_2|$.

In particular, if the interval $X_1 \supset X_2$, and at most only a common endpoint, then $|X_1| > |X_2|$.

Definition 2 (CAI Wen, 1999): let the distance between the point x and the limited real range $X_1 = (a,b)$ is as follows:

$$\rho = (x, X_1) = \left| x - \frac{a+b}{2} \right| - \frac{b-a}{2} = \begin{cases} a - x & x \leq \dfrac{a+b}{2} \\ x - b & x \geq \dfrac{a+b}{2} \end{cases} \quad (1)$$

Nature 2: for a given range of $X = (a,b)$, then:

(1) when $x \in X$, $\rho = (x, X) < 0$ is the necessary and sufficient conditions of $x \neq a, b$;

(2) when $x \notin X$, $\rho = (x, X) > 0$ is the necessary and sufficient conditions of $x \neq a, b$;

(3) $\rho = (x, X) = 0$ is the necessary and sufficient conditions of $x = a$ or $x = b$.

The distance is introduced to accurately describe the location relationship between points and intervals in the quantitative form. When the point is in the range, the classical mathematics assume that the distance between point and interval is zero, while in the extension set, by distance describes the position of point in the range. It realizes the quantitative description that from "category shall be the same" develops until a level of class also having extent differences.

In practical problems, in addition to the need to consider the position between the point and interval, but also has to consider the interval with the interval and consider the position between a point and two ranges.

Definition 3: if $X_1 = (a,b)$, $X_2 = (c,d)$, $X_1 \subset X_2$, x is an any point in real domain, said that

$$D(x, X_1, X_2) = \begin{cases} \rho(x, X_2) - \rho(x, X_1) & \rho(x, X_2) \neq \rho(x, X_1)\, and \quad x \notin X_1 \\ \rho(x, X_2) - \rho(x, X_1) + a - b & \rho(x, X_2) \neq \rho(x, X_1)\, and \quad x \in X_1 \\ a - b & \rho(x, X_2) = \rho(x, X_1) \end{cases} \quad (2)$$

is the bit values of interval sets of x on X_1 and X_2 , denoted by $D(x, X_1, X_2)$.

When the X_1 and X_2 without common endpoints, there

$$D(x, X_1, X_2) = \begin{cases} \rho(x, X_2) - \rho(x, X_1) + a - b & x \in X_1 \\ \rho(x, X_2) - \rho(x, X_1) & x \notin X_1 \end{cases} \quad (3)$$

2.2 Correlation function

Definition 4(CAI Wen, 1998): If $X_1 = (a,b)$, $X_2 = (c,d)$, $X_1 \subset X_2$, X_1 and X_2 without common endpoints, so that

$$k(x) = \begin{cases} \dfrac{\rho(x, X_1)}{D(x, X_1, X_2)} - 1 & \rho(x, X_2) = \rho(x, X_1)\, and \quad x \notin X_1 \\ \dfrac{\rho(x, X_1)}{D(x, X_1, X_2)} & other \end{cases} \quad (4)$$

called $k(x)$ is the elementary correlation function of x on X_1 and X_2 which is obtain the maximum value in the mid-point of X_1 .

Correlation function to enable the calculation of correlation do not have to rely on subjective judgments or statistics, it can quantitatively and objectively describe the elements having the nature or character at a certain extent and the process of quantitative change and qualitative change. This allows correlation function out of bias caused by subjective judgments.

Nature 3: if $X_1 = (a,b)$ and $X_2 = (c,d)$ are two intervals in the real axis, $X_2 \supset X_1$,and no common endpoints, then for any x , has a correlation function:

$$k(x)=\frac{\rho(x,X_1)}{\rho(x,X_2)-\rho(x,X_1)} \tag{5}$$

Nature 4: if $X_1=(a,b)$ and $X_2=(c,d)$ are two intervals in the real axis, $X_2\supset X_1$,and no common endpoints, then for any x to X_1 and X_2, correlation function $k(x)$ has the following property:

（1） $x\in X_1$,and $x\neq a,b\Leftrightarrow k(x)>0$；

（2） $x=a$ or $x=b\Leftrightarrow k(x)=0$；

（3） $x\notin X_1$, $x\in X_2$,and $x\neq a,b,c,d\Leftrightarrow -1<k(x)<0$；

（4） $x=c$ or $x=d\Leftrightarrow k(x)=-1$；

（5） $x\notin X_2$,and $x\neq c,d\Leftrightarrow k(x)<-1$；

（6） $x=\frac{a+b}{2}$, $k(x)$ is the greatest.

Let $\alpha=(a+b)/2$ is the midpoint of X_1, $\beta=(c+d)/2$ is the midpoint of X_2, then according to the distance between point x and a limited real range can be:

$$k(x)=\begin{cases}\frac{x-b}{b-d} & x>\alpha>\beta \quad or \quad x>\beta>\alpha\\ \frac{x-b}{b+c-2x} & \alpha<x<\beta\\ \frac{a-x}{2x-(a+d)} & \beta<x<\alpha\\ \frac{a-x}{c-a} & x<\alpha<\beta \quad or \quad x<\beta<\alpha\end{cases} \tag{6}$$

Correlation function can be determined based on the required value scope $X_1=(a,b)$ and the qualitative change interval $X_2=(c,d)$ of things on certain characteristics, according to set the four points, you can create correlation function, which has an important role for the quantitative research of extension transformation of extension set.

2.3 Correlation function standardization

In practice, matter-element of problems are often multi-dimensional, multi-attribute, multi-object, therefore, an object may have several correlation functions, based on the same premise of the domain and changing conditions, these correlation functions would have a comparable. According to the principle of judging logic operation of correlation functions can be achieved.

"OR" operator: $k_1\vee k_2=\max\{k_1,k_2\}$,it is said that the maximum principle of a number of correlation functions of an object.

"AND" operator: $k_1\wedge k_2=\min\{k_1,k_2\}$,it is said that the minimum principle of a number of correlation functions of an object.

Then, based on the logic operation standard correlation function of an object as follows:

$$k = \frac{k_i}{\max\limits_{1\le i\le n}|k_i|} \tag{7}$$

$-1 \le k \le 1$, $(i = 1, 2, \cdots, n)$.

3. CORRESPONDENCE OF EXTENSION ASSOCIATED

Nature 5: if $X_1 = (a, b)$ is the interval in the real axis, $X_1 \supset x$, then for any x, there is correlation function:

$$K(x) = \frac{-\rho(x, X_1)}{|X_1|} \tag{8}$$

Let $\alpha = (a+b)/2$ is the mid-point of X_1, then:

$$K(x) = \begin{cases} \dfrac{b-x}{b-a} & x > \alpha \\ 0.5 & x = \alpha \\ \dfrac{x-a}{b-a} & x < \alpha \end{cases} \tag{9}$$

Inference 1: When $X_2 \supset X_1 \supset x$, there must be a corresponding point $\pi = 2\alpha - x$, x and π are symmetric on α.

Proof:

(1) When $x \ge \alpha$, there must be $\pi < \alpha$; When $x < \alpha$, there must be $\pi \ge \alpha$, $\pi = 2\alpha - x$.

So, when $X_2 \supset X_1 \supset x$, there must be a corresponding point $\pi = 2\alpha - x$.

(2) $|x-\alpha|$ is the distance of x to α, $|\pi - \alpha|$ is the distance of the corresponding point π to α.

Let $|x-\alpha| = |\pi - \alpha|$

When $x > \alpha$, there must be a corresponding point $\pi < \alpha$, then: $x - \alpha = \alpha - \pi$,

$\pi = 2\alpha - x$ substituted into the above formula will get:

$x - \alpha = \alpha - (2\alpha - x) \Rightarrow x - \alpha = x - \alpha$,

Similarly, when $x < \alpha$, there must be a corresponding point $\pi = 2\alpha - x$, then $\alpha - x = \alpha - x$.

So, when $X_2 \supset X_1 \supset x$, there must be a corresponding point $\pi = 2\alpha - x$, x and π are symmetric on α.

Inference 2: When $X_2 \supset X_1 \supset x$, $x = a$ or $x = b$, there is minimal correlation

$K(x) = K(\pi) = 0$; when $x = \alpha$, π is the corresponding point of X , and has the greatest correlation $K(x) = K(\pi) = 0.5$.

Proof:

(1) When $X_2 \supset X_1 \supset x$ and $x = a$, obtain $x < \alpha$, then:

$$K(a) = \frac{x-a}{b-a} = \frac{0}{b-a} = 0$$

When $X_2 \supset X_1 \supset x$, there must be a corresponding point $\pi = 2\alpha - x$,then $\pi = 2\alpha - x = b$.

Similarly, when $x = b$, $x > \alpha$,then $K(b) = 0$, $\pi = a$.

So, when $X_2 \supset X_1 \supset x$, $x = a$ or $x = b$, there is minimal correlation $K(x) = K(\pi) = 0$.

(2) Let an arbitrary point $a < x < \alpha$, according to the title need to prove that $0 < K(x) < 0.5$, then

Since $b > x > a$, get $K(x) = \dfrac{x-a}{b-a} > 0$,

Based on $K(x) = \dfrac{x-a}{b-a} < 0.5 \Rightarrow \dfrac{x-a}{b-a} - \dfrac{1}{2} < 0$,then $\dfrac{2x-(a+b)}{2(b-a)} = \dfrac{x-\alpha}{b-a}$.

Since $x < \alpha$,then $\dfrac{x-\alpha}{b-a} < 0$ 。

Similarly, when any point $\alpha < x < b$, there exists $0 < K(x) < 0.5$.

When $x = \alpha$, to obtain $\pi = a$ under $\pi = 2\alpha - x$.

Therefore, when $x = \alpha$, π is the corresponding point of X , and has the greatest correlation $K(x) = K(\pi) = 0.5$.

Nature 6: if $X_1 = (a,b)$ and $X_2 = (c,d)$ are two intervals in the real axis, $X_2 \supset X_1$,and no common endpoints, then for any X , has a correlation function $k(x)$, set $\alpha = (a+b)/2$ is the mid-point of X_1 , $\beta = (c+d)/2$ is the mid-point of X_2 , then:

$$k(x) = \begin{cases} \dfrac{x-b}{b-d} & x > \alpha > \beta \quad or \quad x > \beta > \alpha \\ \dfrac{x-b}{b+c-2x} & \alpha < x < \beta \\ \dfrac{a-x}{2x-(a+d)} & \beta < x < \alpha \\ \dfrac{a-x}{c-a} & x < \alpha < \beta \quad or \quad x < \beta < \alpha \end{cases} \tag{10}$$

As the distance between point X and the limited real range X is relative distance, then on the other side of X will inevitably be a corresponding point π , and there is $K_X(x) = K_X(\pi)$.

Inference 3: if $X_1 = (a,b)$ and $X_2 = (c,d)$ are two intervals in the real axis, $X_2 \supset X_1$,and no common endpoints, then for any $X, x \notin X_1, x \in X_2$, has a correlation function $K(x)$, there must exists an association corresponding point π to meet $K_X(x) = K_X(\pi)$.

(1) When $\alpha < x < \beta$, there exists a corresponding point $c < \pi < a$, $\pi = \dfrac{a(c-x)+c(b-x)}{b+c-2x}$

(2) When $\beta < x < \alpha$, there exists a corresponding point $b < \pi < d$, $\pi = \dfrac{b(x-d)+d(x-a)}{2x-(a+d)}$

(3) When $x > \alpha$, $x > \beta$ and $a \le \beta$, there exists a corresponding point $c < \pi < a$, $\pi = \dfrac{a(x-d)+c(b-x)}{b-d}$

(4) When $x > \alpha$, $x > \beta$, $a > \beta$ and $b < x < \dfrac{b(d-c)+d(2a-(c+d))}{2(a-c)}$, there exists a corresponding point $\beta < \pi < a$, $\pi = \dfrac{a(x-d)+d(x-b)}{2x-(b+d)}$; When $x > \alpha$, $x > \beta$, $a > \beta$ and $\dfrac{b(d-c)+d(2a-(c+d))}{2(a-c)} < x < d$, there exists a corresponding point $c < \pi < \beta, \pi = \dfrac{c(x-b)+a(d-x)}{d-b}$;

(5) When $x < \alpha$, $x < \beta$ and $b \ge \beta$, there exists a corresponding point $b < \pi < d, \pi = \dfrac{b(c-x)+d(x-a)}{c-a}$;

(6) When $x < \alpha$, $x < \beta$, $b < \beta$ and $\dfrac{a(d-c)+c(c+d-2b)}{2(d-b)} < x < a$, there exists a corresponding point $b < \pi < \beta$, $\pi = \dfrac{c(a-x)+b(c-x)}{c+a-2x}$; When $x < \alpha$, $x < \beta$, $b < \beta$ and $c < x < \dfrac{a(d-c)+c(c+d-2b)}{2(d-b)}$, there exists a corresponding point $\pi = \dfrac{b(c-x)+d(x-a)}{c-a}$

Proof (1):

According to the known conditions $K_X(x) = K_X(\pi)$, since $\alpha < x < \beta$, $c < \pi < a$, then $\dfrac{x-b}{b+c-2x} = \dfrac{a-\pi}{c-a}$

$$\pi = \frac{a(c-x)+c(b-x)}{b+c-2x}$$

When $\pi > c$, we can get $\frac{a(c-x)+c(b-x)}{b+c-2x} > c$

Because $a(c-x)+c(b-x)<0$ and $b+c-2x<0$,then

$a(c-x)+c(b-x)<c(b+c-2x)$, $(a-c)(c-x)<0$

Because $(a-c)>0$ and $(c-x)<0$, then $\pi > c$.

When $\pi < a$, we can get $\frac{a(c-x)+c(b-x)}{b+c-2x} < a$

Because $a(c-x)+c(b-x)<0$ and $b+c-2x<0$,then $(b-x)(c-a)>0$

Because $(b-x)<0$ and $(c-a)<0$, then $\pi < a$.

Therefore, when $\alpha < x < \beta$, there exists a corresponding point $c < \pi < a$.

The same reason, we can prove that (2) and (3)

Proof (4):

Since $x > \alpha > \beta$, so the corresponding point of x may be $\beta < \pi < a$ or $c < \pi < \beta$.

When $\beta < \pi < a$, according to the known conditions $K_X(x) = K_X(\pi)$, then $\frac{a-\pi}{2\pi-(a+d)} = \frac{x-b}{b-d}$,

we can get $x = \frac{d(\pi-a)+b(\pi-d)}{2\pi-(a+d)}$, $\pi = \frac{a(x-d)+d(x-b)}{2x-(b+d)}$,

if $\beta < \pi < a$, then $b < x < \frac{b(d-c)+d(2a-(c+d))}{2(a-c)}$.

When $c < \pi < \beta$, then $\frac{a-\pi}{c-a} = \frac{x-b}{b-d}$

we can get $x = \frac{b(c-\pi)+d(\pi-a)}{c-a}$, $\pi = \frac{c(x-b)+a(d-x)}{d-b}$,

if $c < \pi < \beta$, then $\frac{b(d-c)+d(2a-(c+d))}{2(a-c)} < x < d$

Therefore, $$\begin{cases} \pi = \frac{a(x-d)+d(x-b)}{2x-(a+d)} & b < x < \frac{b(d-c)+d(2a-(c+d))}{2(a-c)} \\ \pi = \frac{c(x-b)+a(d-x)}{d-b} & \frac{b(d-c)+d(2a-(c+d))}{2(a-c)} < x < d \end{cases}$$

4. EXTENSION GROUP DECISION-MAKING INTERVAL BASED ON ASSOCIATED CORRESPONDENCE UNDER UNCERTAINTY

4.1 Correlation range based on the correspondence

Nature 7: if $X_1 = (a,b)$ is an intervals in the real axis, $X_1 \supset x$, for any x, has a correlation function $k(x)$, set $\alpha = (a+b)/2$ is the mid-point of X_1, then the maximum correlation range $k(x)$ of x on the X_1 as follows:

$$k(x) = \begin{cases} 0 & x = a \quad or \quad x = b \\ 0.5 & x = \alpha \end{cases} \tag{11}$$

Nature 8: if $X_1 = (a,b)$ and $X_2 = (c,d)$ are two intervals in the real axis, $X_2 \supset X_1$,and no common endpoints, then for any x, has a correlation function $K(x)$, set $\alpha = (a+b)/2$ is the mid-point of X_1, $\beta = (c+d)/2$ is the mid-point of X_2, then when $x = \alpha$, there are correlation interval as follows:

$$K(x) = \begin{cases} \dfrac{a-b}{b-a+2c-2x} & x = \alpha < \beta \\ \dfrac{a-b}{b-a+2x-2d} & x = \alpha > \beta \end{cases} \tag{12}$$

Then when $x = \beta$, there are correlation interval as follows:

$$K(x) = \begin{cases} \dfrac{2a-2x}{2a-2x+d-c} & x = \beta < \alpha \\ \dfrac{2x-2b}{2x-2b+d-c} & x = \beta > \alpha \end{cases} \tag{13}$$

When x and interval endpoint a,b,c,d overlap , then correlation degree as follows:

$$K(x) = \begin{cases} \dfrac{a-x}{c-a} & x = c \\ \dfrac{x-b}{b-d} & x = d \\ 0 & x = a \quad or \quad x = b \end{cases} \tag{14}$$

4.2 Extension group decision-making interval based on associated correspondence under missing data

Let $a_i = (a_1, a_2, \cdots, a_n)$ means n schemes , $c_j = \{c_1, c_2, \cdots, c_m\}$ means m decision-makers of a_i , the value of a_i is $c_j(a_i) = (u_{i1}, u_{i2}, \cdots, u_{im})$. Then the composite matter-element of multi-dimensional group decision-making is $a_i = (N, c_j, u_{ij})$. $i = 1,2,\cdots n$, $j = 1,2,\cdots m$.

Let $a = \{a | a_i = (N, c_j, u_{ij}) \in \Re, u_{ij} \in U\}$ is the composite element set of group decision-making, $\hat{A} = \{(u_{ij}, y) | u_{ij} \in U, y = k(u_{ij})\}$ is the extension set, then a matter-element extension set of group decision making in $\Re$ is as follows:

$$\hat{A} = \{(a, y) | a = (N, c_j, u_{ij}) \in \Re, y = K(a_i), y' = T_K K(T_u(a_i))\}$$

Among them, $v_{pj} = [a_{pj}, b_{pj}]$ is the joint field of the matter-element extension set, $v_{lj} = [a_{lj}, b_{lj}]$ is the classical field of the matter-element extension set, $v_{lj} \in v_{pj}$, g indicated that the number of classical field classification, $(g = 1,2,\cdots)$.

$k_j(u_{ij})$ means the association degree between value and interval of assessment, thus, the integrated association degree based on weights χ_j of decision-maker c_j is:

$$K_i(a_i) = \frac{1}{\max\limits_{j=1}^{m} |k_{ij}(u_{ij})|} \sum_{j=1}^{m} (\chi_j \cdot k_{ij}(u_{ij}))$$

$$= \frac{\chi_1 k_1(u_{i1}) + \chi_2 k_2(u_{i2}) + \cdots + \chi_j k_j(u_{ij})}{\max\limits_{1 \le j \le m} |\chi_1 k_1(u_{i1}), \chi_2 k_2(u_{i2}), \cdots, \chi_j k_j(u_{ij})|} \tag{15}$$

If you use $\not{u}_{ij}$ to represent the individual values of missing or wrong, on the basis of the known weights χ_j of decision-makers c_j , then the integrated relational degree as follows:

$$K_i(a_i) = \frac{\chi_1 k_1(u_{i1}) + \chi_2 k_2(\not{u}_{i2}) + \cdots + \chi_j k_j(u_{ij})}{\max\limits_{1 \le j \le m} |\chi_1 k_1(u_{i1}), \chi_2 k_2(\not{u}_{i2}), \cdots, \chi_j k_j(u_{ij})|} \tag{16}$$

Since $d_g(K_i(a_i)) = \begin{cases} l & K_i(a_i) \ge 0 \\ \varnothing & K_i(a_i) < 0 \end{cases}$, then, Definition 5 can be obtained as follows :

Definition 5: $\hat{A}$ is the composite element set of the matter-element of group decision-making $a_i = (N, c_j, u_{ij})$, if there are missing or uncertain values of $\not{u}_{ij}$, when the arbitrary $\not{u}_{ij} \in v_{pj}$, there is $K_i(a_i) < 0$, then $d_g(K_i(a_i)) = \varnothing$, for any $\not{u}_{ij}$, the evaluation that program a_i does not belong to l has no affect; when the arbitrary $\not{u}_{ij} \in v_{pj}$, there is $K_i(a_i) \ge 0$, then $d_g(K_i(a_i)) = l$, for any $\not{u}_{ij}$, the evaluation that program a_i belongs to l has no affect.

Inference 4: If there is missing or uncertain value $\not{u}_{ij}$ in extension group decision-making set $\hat{A}$, given the conditions $K_i(a_i) < 0$, $d_g(K_i(a_i)) = \varnothing$, if $\not{u}_{ij} = \vec{u}_{ij} \in v_{pj}$, to obtain $K_i(a_i) \ge 0$, $d_g(K_i(a_i)) = l$, then the point $\vec{u}_{ij}$ as the boundary points to $\not{u}_{ij}$ on l . Then:

$$d_g(K_i(a_i)) = \begin{cases} l & \underline{u}_{ij} \geq \vec{u}_{ij} \\ \varnothing & \underline{u}_{ij} < \vec{u}_{ij} \end{cases} \tag{17}$$

Set the classical field v_{lj} is l, and $K_i(a_i) \geq 0$, $d_g(K_i(a_i)) = l$, if $\underline{u}_{ij} = \bar{u}_{ij} \in v_{pj}$, when the classical field v_{lj} is $l+1$, there is $K_i(a_i) \geq 0$, $d_{g+1}(K_i(a_i)) = l+1$, then the point $\bar{u}_{ij}$ is the separation points to $\underline{u}_{ij}$ about l and $l+1$. Then:

$$d_g(K_i(a_i)) = \begin{cases} l & \underline{u}_{ij} < \bar{u}_{ij} \\ l+1 & \underline{u}_{ij} \geq \bar{u}_{ij} \end{cases} \tag{18}$$

Inference 5: if $\vec{u}_{ij}$ is the boundary points to $\underline{u}_{ij}$ about l, there must be a corresponding points $\vec{\pi}_{ij}$, makes $k_i(\vec{u}_{ij}) = k_i(\vec{\pi}_{ij})$, then $[\vec{u}_{ij}, \vec{\pi}_{ij}]$ is the interval for $d_g(K_i(a_i)) = l$, $[a_{pj}, \vec{u}_{ij}]$ and $[\vec{\pi}_{ij}, b_{pj}]$ are the intervals for $d_g(K_i(a_i)) = \varnothing$.

Inference 6: if $\bar{u}_{ij}$ is the separation points to $\underline{u}_{ij}$ about l and $l+1$, there must be a corresponding points $\bar{\pi}_{ij}$, makes $k_i(\bar{u}_{ij}) = k_i(\bar{\pi}_{ij})$, then $[\bar{u}_{ij}, \bar{\pi}_{ij}]$ is the interval for $d_g(K_i(a_i)) = l+1, [a_{pj}, \bar{u}_{ij}]$ and $[\bar{\pi}_{ij}, b_{pj}]$ are the intervals for $d_g(K_i(a_i)) = l$.

5. THE MODEL OF EXTENSION GROUP DECISION-MAKING BASED ON ASSOCIATED CORRESPONDENCE UNDER UNCERTAINTY

5.1 Purpose and ideas

The treatment method of missing data based on the extension correspondence is a kind of reverse thinking, with the correlation to predict the possible scope and choice for the missing data under the purpose and the certain constraint condition.. This method can not only take the advantages of point estimates of normal order, but also highlight the merits of interval estimation of reversed order.

The flow of missing data processing based on the extension correspondence mainly includes three parts: the first part is extension transformation, which applies the extension transformation to first evaluating the programs characterized by partial absence of value by setting the basic extension matter-element, extension set and judge set; the second part is the interval estimate, which is through correlation analysis to analyze demarcation point and class point of programs and estimate the potential scope of the missing data and the condition for missing data processing; the third part is a dynamic analysis, which is according to the effect of compensate for missing values, to observe possible changes of the program evaluation based on changing the evaluation value, the evaluation field and weights and adjust and improve missing value so as to enhance the estimated effect.

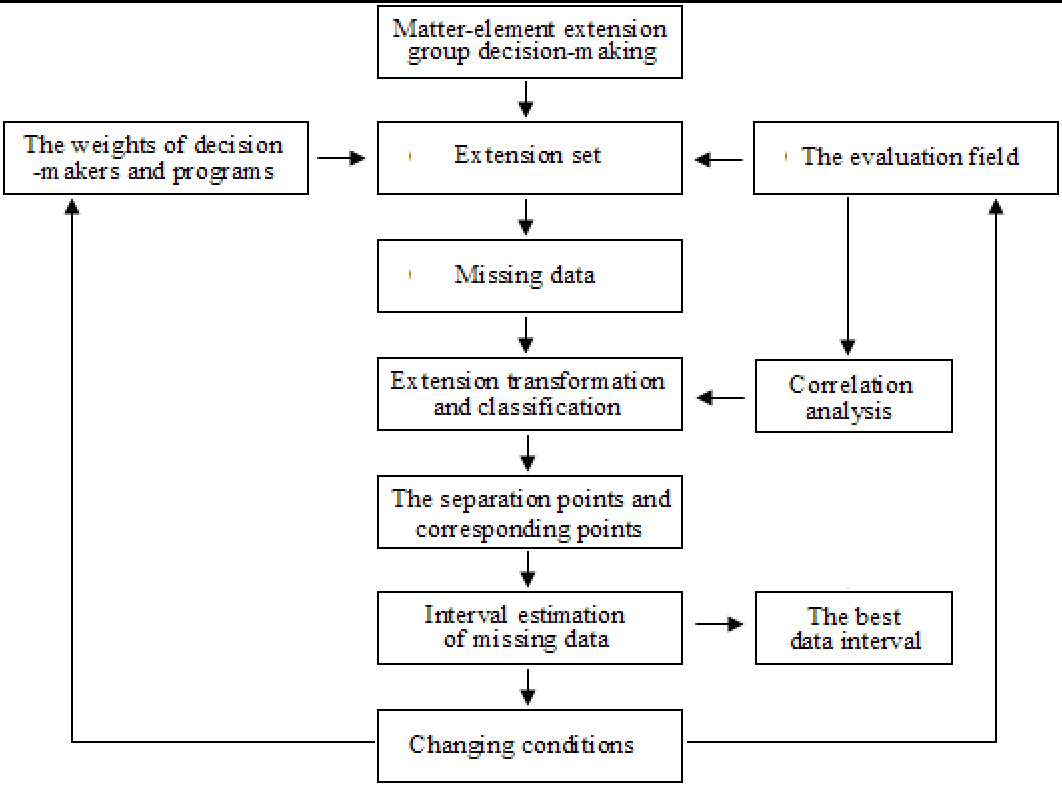

Figure 1: Extension group decision thinking based on correspondence under data missing

5.2 Steps

① To build composite matter-element of extension group decision-making $a_i = (N, c_j, u_{ij})$ and extension group decision-making set $\hat{A}$, set the weight χ_j of policy makers c_j;

② To input evaluation values $c_j(a_i)$ of to policy makers c_j about program a_i, building the matter-element matrix of extension group decision-making;

③ To set joint field $v_{pj} = [a_{pj}, b_{pj}]$ and the classical field $v_{lj} = [a_{lj}, b_{lj}]$, and analysis whether the data is missing or exceed the festival field in $a_i = (N, c_j, u_{ij})$;

④ To remove uncertainty values, calculated correlation degree $k_j(u_{ij})$ from each policy makers c_j to program a_i on v_{pj} and v_{lj};

⑤ To calculate $K_i(a_i)$, by Definition 5, for any u_{ij}, to determine that program a_i whether belonging to l, if no effect then go to step ⑧.

⑥According to Inference 6, to determine whether there is a boundary point $\vec{u}_{ij}$, if there is a critical point, then calculate the corresponding point $\vec{\pi}_{ij}$, to determine $[\vec{u}_{ij}, \vec{\pi}_{ij}]$ is the interval for $d_g(K_i(a_i)) = l$, if there is no boundary point, then turned step ⑧;

⑦ To determine whether there is a separation point $\bar{u}_{ij}$, if there is a separation point to make $d_g = l$, then calculate the corresponding point $\bar{\pi}_{ij}$, to determine $[a_{pj}, \bar{u}_{ij}]$ and $[\bar{\pi}_{ij}, b_{pj}]$ are intervals for $d_g(K_i(a_i)) = l$, if there is no boundary point, then turned step ⑧; if there is a separation point to make $d_g = l+1$, then calculate the corresponding point $\bar{\pi}_{ij}$, to determine $[\bar{u}_{ij}, \bar{\pi}_{ij}]$ is the interval for $d_g(K_i(a_i)) = l+1$, and turned step⑧, or direct go to step ⑧;

⑧ To estimate the range of u_{ij};

⑨ If u_{ij} can meet the requirements of decision-making, go to step ⑩, otherwise go to step①; if you do not need updating weights, go to step ②; if you do not need to update data, then go to step③;

⑩ End.

6. CONCLUSION

The present decision-making problems are often ones under uncertainty which are a kind of dynamic, complex and correlated decision-making process. Due to the complexity of the decision-making problem, the multi-hierarchy of decision-making process, the difference methods in data processing it will lead to the absence of decision-making data, error or ambiguity, further affecting the decision making. In terms of the features of missing data and the requirements of data processing in extension group decision-making, combined correlation function with extension transformation, the extension correspondence is applied to analyze the possible scope and range of decision-making data from the reverse perspective. It not only reduces the decision-making bias for subjective estimation, but also improves the results of data forecasting and analysis and provides a basis for recognition, judging and selection extension group decision-making under uncertainty.

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