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The Methods to Trisect an Acute Angle

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Abstract

This paper introduces the condition required to trisect an angle. The authors believe that, if you want to trisect the exterior angle of a triangle, the measures of two remote angles must have a proportion of 2:1. The deduction says that such triangle can be divided into two smaller isosceles triangles. One base angle of the first smaller isosceles triangle is the exterior angle of the other triangle. Construct this triangle with measures of the remote angles having a proportion of 2:1. There is a ray being the side of one isosceles triangle, as well as the base line of the other. The authors provide another method and predicts even more.

Key words: Internal angle; Ray; Angle trisection; Triangle; Characteristic supplementary angle triangle; Characteristic quadrangle; Isosceles triangle

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INTRODUCTION

The angle trisection problem is the first one of the three major problems in ancient Greek geometry[1]. Hippias and Nicomdes, the mathematicians of ancient Greek left the problem unresolved[2], that is to trisect an angle with just compass and straightedge. But their efforts and contributions are admired and respected world-wide.

Now there are two definitely opposing opinions on it: One is Pierre Wantzel's proof in 1837[3] that claimed, it was impossible to trisect an angle by only compass and straight-edge; Wikipedia[4] also claims that, "it is generally impossible to trisect an angle". The other is such scholarly websites as ScholarOne's embodying of "angle trisection" and related papers, as well as in SCI. Hitherto there is no confute of its conclusion from authorities, so it is supported practically.

According to the "exterior angle theorem"[5]: "the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote angles." If we construct a triangle inside the supplementary angle, with the proportion of remote angles which are not adjacent to the vertex angle of the exterior angle to be 2:1, the trisection can be successful.

The authors worked out such triangle with remote angle proportion of 2:1, and accomplished the first instance of angle trisection. Dear mathematicians, please review the manuscript impartially and strictly, and figure out errors in our construction method (please indicate the specific location if possible). You will receive the sincerest appreciation and reverence from the authors! So that we can make improvements and continue with the study!

1. THE CHARACTERISTIC

[1] [published in China]Continued - Geometry Dictionary, [Japan] Nagasawa Saburo, compiled by XUE De-Jiong, WU Zai-Yao.

[2] Same as above, p.518
[3] Jour . de Math, 2, 1937, 366-372.
[4] Angle trisection.Wikipedia.[4/9/2018].https://en.wikipedia.org/wiki/Angle_trisection
[5] Exterior angle theorem.Wikipedia.[4/9/2018] .https://en.wikipedia.org/wiki/Exterior_angle_theorem

SUPPLEMENTARY ANGLE TRIANGLE

According to the "exterior angle theorem", construct a triangle inside the supplementary angle of the given exterior angle, so that the proportion of its remote interior angles is 2:1. Then we can trisect the exterior angle of the triangle with the supplementary angle being one of its vertex angles.

We construct two isosceles triangles as collocation, let one base angle of the thicker one be the exterior angle of the thinner one. Because the base angles of the thinner isosceles triangle are equal, the sum of the two base angles equals to the measure of a base angle of the thicker isosceles triangle. So that the two interior angles of the triangle with the supplementary angle being a base angle can have a proportion of 2:1. The authors name the triangle composed by the two isosceles triangle, which is inside the supplementary angle, as a "characteristic supplementary angle triangle".

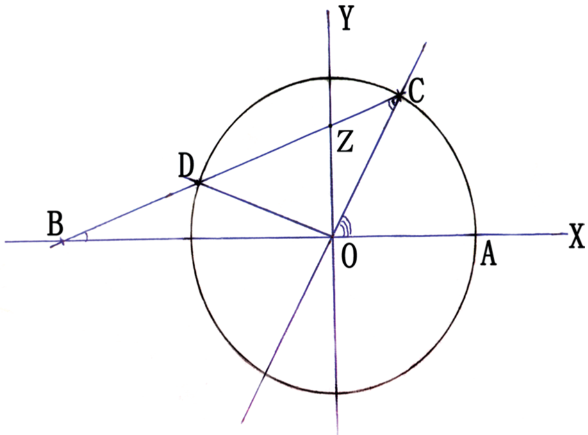

Figure 1
Characteristic Supplementary Angle Triangle (DB=DO)

2. "1, 2, 3 THEOREM"

Suppose the exterior angle given is ∠COA, and DB=DO.

Show that ∠CBO=(1/3)∠COA =δ.

Proof: ∵ Radius DO=CO, ∴∠DCO=∠CDO. ∠CDO is the exterior angle of △DBO, ∴∠CDO =∠CBO+∠DOB. ∵DO=DB, ∠CBO=∠DOB. Therefore ∠BCO(i.e.∠DCO)=∠CDO = 2∠CBO. And ∵∠COA is the exterior angle of the characteristic supplementary angle triangle △CBO. ∴∠COA=∠CBO+2∠CBO=3∠CBO. That is, ∠CBO=(1/3)∠COA =δ.

Because ∠CBO=1δ; ∠BCO=2δ; ∠COA=3δ. ∴We call it: "1,2,3 theorem".

As long as such "characteristic supplementary angle triangle" △BCO can be constructed from the given exterior angle ∠COA, any given exterior angle inside the supplementary angle can be trisected. At least, this theorem indicates that it is possible to trisect an angle by purely geometric method.

Ray CB is really important to the "characteristic supplementary angle triangle" △BCO, it is the base line CD of the thicker isosceles triangle △COD, as well as the side line DB of the thinner isosceles triangle △DBO. ∴we call it the "ray of gold".

3. THE CHARACTERISTIC QUADRANGLE

Provided that a preset angle ∠DBH=δ, construct a face-to-face ∠DOH =∠DBH to its right, forming a thinner isosceles triangle △DBO. Draw a circle taking the vertex O as the center, DO as the radius. Then extend BD to the arc intersecting at point C, connect CO(=DB) to be the radius, construct ∠COA to be triple the measure of the angle. When ∠DBH grows, the exterior angle ∠COA also grows, the radius changes as well. But the proportion of ∠COA to ∠DBH is always 3:1.

For the varied exterior angle ∠COA, a varied "characteristic supplementary angle triangle" can always be construct. But a compactly relevant outer packing to "characteristic supplementary angle triangle" is required--the "characteristic quadrangle". The operation procedure is as follows: Put the exterior angle ∠COA in a X-Y coordinate system, construct CE//AB, then a perpendicular EK from point E, intersecting with the extension line of CO at K. Extend CE to FC=2CO. Construct a circle taking the origin O as the center, CO as the radius. From the point of intersection I with the X-axis, connect IF//EO. Form a rhombus EFIO, then construct FB//CO. A parallelogram CFBO is formed. This can be called the "characteristic quadrangle".

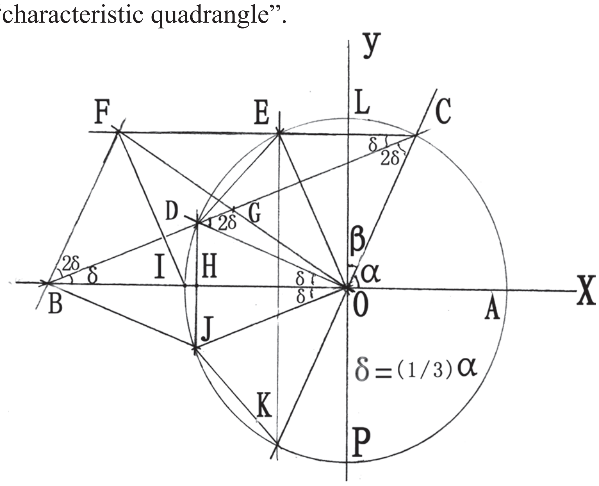

Figure 2
Characteristic Quadrangle

4. ANGLE TRISECTION

The longer diagonal of this the "characteristic quadrangle" is CB, the "ray of gold". The "supplementary angle triangle"△CBO is underneath. According to "1,2,3 theorem", $\angle BCO=2\angle CBO$, $\angle CBO=(1/3)\angle COA=\delta$. The angle trisection is completed.

Range of application: Only an acute angle between 0° and 90° is applicable; While $\angle COA = 0°$, C, A coincides. AB will be 3 times the radius;While $\angle COA=90°$, the characteristic quadrangle becomes the rectangle CFBO, where CF is 2 times the radius, D is at the midpoint of the rectangle diagonals. Still we have $\angle CBO=(1/3)\angle COA=30°$.