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The Statistical Analysis of a Certain Kind of Sales Diffusion Model

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Abstract

The essay works out MLE (Maximum Likelihood Estimation) of probability model parameter about the third sales diffusion curve given by ZHENG Zukang and researches the existence of the estimation. Besides, this essay inspects the accuracy of the estimation via Monte Carlo simulation and throws light on methods of essay by simulated data.

Key words: Sales diffusion model; Maximum Likelihood Estimation; Monte Carlo simulation

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INTRODUCTION

In recent years, many scholars have made research on the problem of new product diffusion. The purport of[1] is the influencing factor of new product diffusion. While[2] predicts new product diffusion by establishing model. In ZHENG Zukang[3] analyzes the process of sales diffusion by the means of establishing model. On the basis of model, it is easy to predict the time of sales peak and sales more than half. "Impulse" in the essay is used to describe the influence of three vital factors of new product diffusion – consumers, technicians, sales force. And it relates "impulse" with sales diffusion curve, average diffusion rate of sale and the time of sale period to establish three models separately, which makes people grasp the process of sales diffusion so enterprises can make new product diffuse in the market successfully.

The essay works out Maximum Likelihood Estimation of probability model parameter about the third sales diffusion curve given by ZHENG Zukang and researches the existence of the estimation. Besides, this essay inspects the accuracy of the estimation via Monte Carlo simulation and throws light on methods of essay via Monte Carlo simulation.

1. THE MODEL AND DIGITAL CHARACTERISTICS

"Impulse" is linked only with the time T in "Model 3" from[3]. Equation is established as follows:

$$\frac{\frac{dF(t)}{dt}}{1-F(t)} = bt + c, t > 0, b > 0, c > 0 \tag{1}$$

Where $F(t)$ is the distribution function of non-negative random variable T. Thus,

$$F(t) = 1 - \exp\left\{-\left(\frac{b}{2}t^2 + ct\right)\right\} \tag{2}$$

If random variable T has distribution function like above, it obeys extended two-parameter Weibull distribution. Its density function and failure rate function are as follows:

$$f(t)=(bt+c)\exp\left\{-\left(\frac{b}{2}t^2+ct\right)\right\},\ \lambda(t)=bt+c$$

$$b,c>0,t>0. \tag{3}$$

The r moments of this distribution:

$$ET^r=2e^{\frac{c^2}{2b}}\int_{\frac{c}{\sqrt{2b}}}^{+\infty}\left(\sqrt{\frac{2}{b}}x-\frac{c}{b}\right)^r xe^{-x^2}dx\cdot \tag{4}$$

2. THE MAXIMUM LIKELIHOOD ESTIMATION OF PARAMETER

Set T_1, T_2, T_3,…, T_n from the overall T of extended two-parameter Weibull distribution, for a sample of size n. The observed value is t_1, t_2,…, t_n, and $T_{(1)}$, $T_{(2)}$,…, $T_{(n)}$ denoted by the order statistics. Besides, its order observed values denoted by $t_{(1)}$, $t_{(2)}$,…, $t_{(n)}$.

Likelihood function:

$$L(b,c)=\prod_{i=1}^{n}(bt_i+c)\exp\left\{-\left(\frac{b}{2}\sum_{i=1}^{n}t_i^2+c\sum_{i=1}^{n}t_i\right)\right\} \tag{5}$$

$$\frac{\partial\ln L(b,c)}{\partial b}=\sum_{i=1}^{n}\frac{t_i}{bt_i+c}-\frac{1}{2}\sum_{i=1}^{n}t_i^2,\ \frac{\partial\ln L(b,c)}{\partial c}=\sum_{i=1}^{n}\frac{1}{bt_i+c}-\sum_{i=1}^{n}t_i$$

Note that $\frac{\partial\ln L(b,c)}{\partial b}=0$, $\frac{\partial\ln L(b,c)}{\partial c}=0$, namely the equations are as follows:

$$\begin{cases}\sum_{i=1}^{n}\frac{t_i}{bt_i+c}-\frac{1}{2}\sum_{i=1}^{n}t_i^2=0\\ \sum_{i=1}^{n}\frac{1}{bt_i+c}-\sum_{i=1}^{n}t_i=0\end{cases}$$

The equations of deformation, then division:

$$\frac{\sum_{i=1}^{n}\frac{t_i}{t_i+c/b}}{\sum_{i=1}^{n}\frac{1}{t_i+c/b}}=\frac{\sum_{i=1}^{n}t_i^2}{2\sum_{i=1}^{n}t_i} \tag{6}$$

Note $\beta=\frac{c}{b}$, above equation is as follows:

$$\frac{\sum_{i=1}^{n}\frac{t_i}{t_i+\beta}}{\sum_{i=1}^{n}\frac{1}{t_i+\beta}}=\frac{\sum_{i=1}^{n}t_i^2}{2\sum_{i=1}^{n}t_i} \tag{7}$$

Lemma: if $\beta>0$, $t_i>0$, $i=1,2,\dots,n$ satisfy the following equation: $\frac{n}{\sum_{i=1}^{n}1/t_i}<\frac{\sum_{i=1}^{n}t_i^2}{2\sum_{i=1}^{n}t_i}<\frac{\sum_{i=1}^{n}t_i}{n}$. The equation of β: $\frac{\sum_{i=1}^{n}\frac{t_i}{t_i+\beta}}{\sum_{i=1}^{n}\frac{1}{t_i+\beta}}=\frac{\sum_{i=1}^{n}t_i^2}{2\sum_{i=1}^{n}t_i}$ has the unique positive root.

Prove: note that $g(\beta)=\frac{\sum_{i=1}^{n}\frac{t_i}{t_i+\beta}}{\sum_{i=1}^{n}\frac{1}{t_i+\beta}},\beta>0$ (8)

$$\lim_{\beta\to 0}g(\beta)=\frac{n}{\sum_{i=1}^{n}1/t_i},\ \lim_{\beta\to+\infty}g(\beta)=\lim_{\beta\to+\infty}\frac{\sum_{i=1}^{n}\frac{t_i\beta}{t_i+\beta}}{\sum_{i=1}^{n}\frac{\beta}{t_i+\beta}}=$$

$$\lim_{\beta\to+\infty}\frac{\sum_{i=1}^{n}\frac{t_i}{t_i/\beta+1}}{\sum_{i=1}^{n}\frac{1}{t_i/\beta+1}}=\frac{\sum_{i=1}^{n}t_i}{n}$$

$$g'(\beta)=\frac{\sum_{i=1}^{n}\frac{t_i}{t_i+\beta}\sum_{i=1}^{n}\frac{1}{(t_i+\beta)^2}-\sum_{i=1}^{n}\frac{t_i}{(t_i+\beta)^2}\sum_{i=1}^{n}\frac{1}{t_i+\beta}}{\left(\sum_{i=1}^{n}\frac{1}{t_i+\beta}\right)^2}$$

$$g_1(\beta)=\sum_{i=1}^{n}\frac{t_i}{t_i+\beta}\sum_{j=1}^{n}\frac{1}{(t_j+\beta)^2}-\sum_{i=1}^{n}\frac{t_i}{(t_i+\beta)^2}$$

Where $\sum_{j=1}^{n}\frac{1}{t_j+\beta},\beta>0$

Meanwhile

$$g_1(\beta)=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\frac{t_i}{t_i+\beta}\cdot\frac{1}{(t_j+\beta)^2}+\frac{t_j}{t_j+\beta}\cdot\frac{1}{(t_i+\beta)^2}-\frac{t_i}{(t_i+\beta)^2}\cdot\frac{1}{t_j+\beta}-\frac{t_j}{(t_j+\beta)^2}\cdot\frac{1}{t_i+\beta}\right]$$

$$=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{(t_i+\beta)^2(t_j+\beta)^2}\begin{pmatrix}t_i^2+\beta t_i+t_i^2+\beta t_j\\ -t_it_j-\beta t_i-t_it_j-\beta t_j\end{pmatrix}$$

$$=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(t_i-t_j)^2}{(t_i+\beta)^2(t_j+\beta)^2}>0$$

Namely, $g'(\beta)>0$. $g(\beta)$ is the monotone increasing function of β, so lemma is right.

If sample values satisfy $\frac{n}{\sum_{i=1}^{n} 1/t_i} < \frac{\sum_{i=1}^{n} t_i^2}{2\sum_{i=1}^{n} t_i} < \frac{\sum_{i=1}^{n} t_i}{n}$. Hence, maximum likelihood estimation of b, c is $\hat{b} = \frac{\sum_{i=1}^{n} \frac{1}{t_i+\hat{\beta}}}{\sum_{i=1}^{n} t_i}, \hat{c} = b\hat{\beta}$.

We produce one thousand group sample values t_i, $i = 1, 2, \cdots, n$ via Monte Carlo simulation for different true value of n, b, c. The numbers of sample values group which satisfy the existence condition of maximum likelihood estimation, namely the condition of Lemma are as Table 1. From the Table 1, it can be seen that about 76% satisfy the existence condition of maximum likelihood estimation in one thousand groups. Maximum Likelihood Estimation of parameters exists mostly for the third model of sales diffusion.

Table1
The Number of Group Satisfy Condition

n	*b* True value	*c* True value	*β* True value	The number of group
10	1	1	1	812
	1	2	2	798
	1	3	3	765
	2	1	0.5	782
	2	2	1	792
	2	3	1.5	773
15	1	1	1	878
	1	2	2	848
	1	3	3	770
	2	1	0.5	882
	2	2	1	881
	2	3	1.5	833
20	1	1	1	941
	1	2	2	852
	1	3	3	778
	2	1	0.5	938
	2	2	1	929
	2	3	1.5	833

Table 2
The Accuracy of Maximum Likelihood Estimation Estimation

n	*c*	*b*	*β*	$\hat{\beta}$		$\hat{c}$		$\hat{b}$	
	True value	True value	True value	Mean	Mean square error	Mean	Mean square error	Mean	Mean square error
15	0.375	0.5	0.75	0.7415	0.1037	0.3604	0.0241	0.5805	0.0537
	0.75	1	0.75	0.717	0.1391	0.7402	0.0758	1.1933	0.2379
	0.5	0.5	1	0.8875	0.1982	0.4799	0.0301	0.6163	0.0611
	0.75	0.75	1	1.0038	0.1818	0.7552	0.0547	0.8452	0.1151
	1	1	1	0.9898	0.1755	1.0254	0.0971	1.1634	0.2164
	0.875	0.5	1.75	1.4539	0.2377	0.8585	0.0652	0.6353	0.0622
20	0.375	0.5	0.75	0.8289	0.0868	0.402	0.0146	0.5154	0.0249
	0.75	1	0.75	0.7181	0.0989	0.7353	0.0483	1.1473	0.167
	0.5	0.5	1	1.0514	0.1643	0.5149	0.0219	0.5315	0.0288
	0.75	0.75	1	1.0319	0.1607	0.768	0.0458	0.8227	0.0786
	1	1	1	0.9453	0.174	0.9813	0.0797	1.1795	0.2064
	0.875	0.5	1.75	1.6656	0.2377	0.8808	0.05	0.5616	0.0332

Next, we inspect the accuracy of the estimation via Monte Carlo simulation. We calculate mean and mean square error for one thousand group sample values t_i, $i = 1, 2, \cdots, n$ which satisfy the existence condition of maximum likelihood estimation. Results are as Table 2.

From Table 2, it can be seen that the accuracy of maximum likelihood estimation is satisfactory. And mean square error decreases with the sample size increases.

Example: Supposing that the capacity of sample $n = 20$, the true value of parameters is $c = 1.75$, $b = 1$, then $\beta = 1.75$. We generate a set of random number which obeys extended two-parameter Weibull distribution via Monte Carlo simulation are as follows:

0.6107, 0.1148, 0.1483, 0.0774, 0.3075, 0.7558, 1.8253, 0.5652, 0.7637, 0.3950

0.0531, 0.5035, 0.8574, 0.0352, 0.1941, 0.7109, 0.2527, 0.2855, 0.2411, 0.8486

After computing:

$$\frac{\sum_{i=1}^{n} t_i}{n} = 0.4773, \quad \frac{\sum_{i=1}^{n} t_i^2}{2\sum_{i=1}^{n} t_i} = 0.4154, \quad \frac{n}{\sum_{i=1}^{n} 1/t_i} = 0.1810,$$

which satisfy the existence condition of Maximum likelihood estimation:

$$\frac{n}{\sum_{i=1}^{n} 1/t_i} < \frac{\sum_{i=1}^{n} t_i^2}{2\sum_{i=1}^{n} t_i} < \frac{\sum_{i=1}^{n} t_i}{n}$$

, Make use of the method of this essay: $\hat{c} = 1.6904$, $\hat{b} = 0.9747$, $\hat{\beta} = 1.7342$.

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