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Modeling Memorization and Forgetfulness Using Differential Equations

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Abstract: Research Context: The aim of the study was to use differential equations to model memorization of students based on a given data taking into account forgetfulness.
Research Methods: The purpose of this paper was to decipher the rate at which students memorized the stuff that required memorization in the area of axioms and proofs of theorems as well as considering the fact that they will forget some of them along the way. The usage of differential equation was employed to model the trend. The paper contributes to the literature by documenting that students can memorize large number of stuff even beyond their perceived imaginations.
Conclusion: This study employed the usage of differential equations to model the rate at which students could memorize a given number of axioms and proofs, considering the fact that they will forget some of them along the way. Persons who are able to absorb and retain more are able to recollect better than those who can absorb more and retain less. On the other hand, those who can absorb less and retain more have an upper hand in recollection over those who can absorb more and retain less. Consequently it is better to have a higher retention constant than a higher absorption rate. Factors like

the learning strategy, learning materials, learning environment, study mates have either a positive or negative influence on an individual's absorption and retention in the long term.

Key words: Memorization; Differential equations; Model; Forgetfulness; Absorption rate; Learning environment

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1. INTRODUCTION

A differential equation in Polking *et al.* (2005) is an equation involving an unknown function and its derivatives. Differential equations are a powerful tool in constructing mathematical models for the physical world. Their use in industry and engineering is so widespread and they perform their task so well that they are clearly one of the most successful of modeling tools (Borrelli *et al.*, 2004). Many of the principles, or laws, underlying the behaviour of the natural world are statements or relations involving rates at which things happen. When expressed in mathematical terms the relations are equations and the rates are derivatives. Equations containing derivatives are differential equations. A differential equation that describes some physical process is often called a mathematical model of the process (Boyce *et al.*, 2001). In science, we understand our real world by observations, collecting data, finding rules inside or among them, and eventually, we want to explore the truth behind and to apply it to predict the future. This is how we build up our scientific knowledge. The above rules are usually in terms of mathematics known as mathematical models. One important such models is the ordinary differential equations. It describes relations between variables and their derivatives. Such models appear everywhere. For instance, population dynamics in ecology and biology, mechanics of particles in physics, chemical reaction in chemistry, economics, etc (Chern, 2007). The study was to gain insight into ability to memorize using ordinary differential equations (Obeng-Denteh *et al.*, 2012).

Try to imagine life without memories. We would have no identity. We would ask the same questions over and over because we would not be able to remember the answers to them. We would live forever in the present moment and have no recollection of our pasts, including people and experiences that are important to us, and no anticipation of the future.

While memory is crucial for all of us, there is no time during which memory demands are greater than the school years. The school environment, however, is not often a "memory-friendly" one. Children are presented with new information all throughout the school day and given little opportunity to consolidate new information before more new information is presented to them (Thorne, 2009).

Just like memorization, things learnt can also be forgotten if they are not constantly revised. Contrary to popular belief, being smart is not synonymous to having a good memory or good retention but lies in the lifestyle of a person, attitude, diet and habits (Edwards, 2008). Kay, one of the best authorities on this subject, said "Unless the mind possessed the power of treasuring up and recalling its past experiences, no knowledge of any kind could be acquired" (Atkinson, 2009). Effective memorization occurs when you can make an association between the new and what you already know. As Cowan (1988) puts it, "new information must make contact with the long-term knowledge stored in order for it to be categorically coded" (Klemm, 2007).

Learning involves comprehending the world by reinterpreting knowledge as quoted in Ramsden 1992 (Smith, 2003).The person who actually learns, rather than merely memorizing is not only able to relate existing knowledge and apply it to new situation but more importantly, he can critically judge ideas of learned people (Veksler, 2002).

Just like any muscle, you need to exercise your brain so that it does not deteriorate (Edwards, 2008). Although memory is usually robust and accurate, different disease processes can disrupt memory and cause either distortions or outright failure (Budson *et al.*, 2001). Forgetfulness can stem from many causes, both physical and psychological. Some causes are reversible while others can be managed with medication (Boyd, 2012).

In the light of the above, we seek to use differential equations to model a few of such phenomena namely; memorization and learning taking into account forgetfulness.

2. METHODOLOGY

The study employed a quantitative methodology which was adopted to allow the researchers to gather more precise and quantifiable information on the memorizing abilities of students. A differential equation involving absorption constant and retention rate was used to model the recall ability of students. The rate of retention and the constant of absorption which were assumed to be between zero and one were constantly varied and the amount recalled at different times were recorded using our differential equation model. This is due to the fact that different individuals have different absorption constants and retention rates.

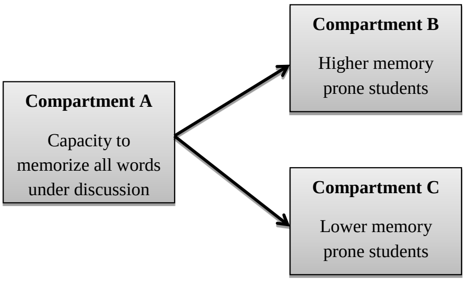

Figure 1
Compartmentalised Systems of Memorization

The following assumptions were made:

- The model considered high achiever and lower achiever in the same classroom.
- Different individuals have different absorption rate.

- Different individuals have different retention.
- Environmental condition(s) in the classroom where student were taught or will be taught is also important.
- Background of the student to be taught affect retention, memorization and forgetfulness.

In the model above, compartment A denotes the capacity to memorize the words under discussion. Compartment B denotes the ability to retain higher amount of words memorized while compartment C represents the capacity to retain a lesser amount of words memorized. In Figure 2, the meaning of the symbols used can be found in 4.1.

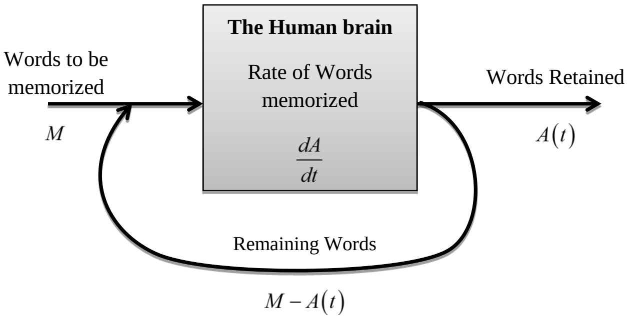

Figure 2
Compartment Illustrating the Amount of Words in and out of the Brain

3. LIMITATIONS OF THE STUDY

Practically, it is virtually impossible to measure an individual's retention constant and absorption rate which made it difficult to determine the accuracy of results obtained. The model does not take into account an individual's physical and mental state hence making our results biased. Above all collation of data would have been financially intensive since all participants would have to be compensated for their time.

4. MODEL FORMULATION AND ANALYSIS

4.1. Model Formulation

Considering a given subject to be memorized, the rate at which it is memorized is assumed to be proportional to the amount left to be memorized (Zill, 2001). Then

$$\frac{dA}{dt} \propto (M - A) \tag{1}$$

which implies that

$$\frac{dA}{dt} = k_1(M - A) \tag{2}$$

where

- k_1 is the factor that affects the rate of absorption,
- $A(t)$ is the amount to be memorized in time t,
- M is the total amount to be memorized, and
- $M - A(t)$ is the amount remaining to be memorized.

However, when forgetfulness is taken into account, the model becomes:

$$\frac{dA}{dt} = k_1 (M - A) - k_2 A \tag{3}$$

In which case k_2 is the forgetfulness factor and where $k_1, k_2 > 0$.
It follows that

$$\frac{dA}{dt} = k_1 M - A (k_1 + k_2) = k_1 M - A k_3$$
$$\frac{dA}{dt} + A k_3 = k_1 M$$

where $k_3 = k_1 + k_2$.

4.2. Methods of Solution

Using the method of integrating factors, the integrating factor is given by

$$e^{\int k_3 dt} = e^{k_3 t}$$

$$e^{k_3 t}\frac{dA}{dt} + e^{k_3 t} A k_3 = e^{k_3 t} k_1 M$$
$$\frac{d}{dt}\left(A e^{k_3 t}\right) = k_1 M e^{k_3 t}$$
$$A e^{k_3 t} = k_1 M \int e^{k_3 t} dt + k_4$$
$$A e^{k_3 t} = \frac{k_1}{k_3} M e^{k_3 t} + k_4$$
$$A(t) = \frac{k_1}{k_3} M + k_4 e^{-k_3 t}$$

The final equation can be rewritten as

$$A(t) = aM + be^{-ct} \tag{4}$$

where $a = \dfrac{k_1}{k_3} = \dfrac{k_1}{k_1 + k_2} > 0$, $c = k_3 = k_1 + k_2 > 0$ and $b > 0$.

Equation (2) would give a similar solution with $k_2 = 0$, thereby making $a = 1$. This means that taking into consideration forgetfulness, it has reduced the proportion of the total amount M to be memorized.

Now solving for the constants using the initial solution, we have:
At $t = 0$, $A(t) = 0$, hence,

$$aM + b = 0$$
$$b = -aM$$

Therefore substituting the value of b into (4), we obtain

$$A(t) = aM - aMe^{-ct} = aM\left(1 - e^{-ct}\right) \tag{5}$$

Let a be the constant of retention and c be the rate of absorption, this means that $t \to \infty$, $A(t) \to aM$.

The explanation to this is that with time, the student can memorize a fraction of the M information available due to the fact that he will forget some of the information learned. In order to solve for the constants a and c, we will need two initial values other than at time $t = 0$. Now we choose two different times t_1 and t_2 such that $t_2 = 2t_1$, thus

$$\begin{aligned} A(t_1) &= aM\left(1 - e^{-ct_1}\right) \quad \text{and} \\ A(t_2) &= aM\left(1 - e^{-ct_2}\right) = aM\left(1 - e^{-2ct_1}\right). \end{aligned}$$

Solving the two equations simultaneously, we have

$$aM = \frac{A(t_1)}{(1 - e^{-ct_1})} = \frac{A(t_2)}{(1 - e^{-2ct_1})}$$

$$\begin{aligned} A(t_1)\left(1 - e^{-2ct_1}\right) &= A(t_2)\left(1 - e^{-ct_1}\right) \\ A(t_1) - A(t_1)e^{-2ct_1} &= A(t_2) - A(t_2)e^{-ct_1} \\ 0 = A(t_1)\left(e^{-ct_1}\right)^2 &- A(t_2)e^{-ct_1} + A(t_2) - A(t_1) \end{aligned}$$

The equation has become a quadratic in terms of e^{-ct_1}

$$e^{-ct_1} = \frac{A(t_2) \pm \sqrt{[A(t_2)]^2 - 4A(t_1)[A(t_2) - A(t_1)]}}{2A(t_1)}$$

The discriminant can also be further simplified to

$$\begin{aligned} &[A(t_2)]^2 - 4A(t_1)[A(t_2) - A(t_1)] \\ =&[A(t_2)]^2 - 4A(t_1)A(t_2) + [2A(t_1)]^2 \\ =&[A(t_2) - 2A(t_1)]^2 \end{aligned}$$

Making the solution

$$\begin{aligned} e^{-ct_1} &= \frac{A(t_2) \pm \sqrt{[A(t_2) - 2A(t_1)]^2}}{2A(t_1)} \\ &= \frac{A(t_2) \pm [A(t_2) - 2A(t_1)]}{2A(t_1)} \end{aligned}$$

Since $e^{-ct_1} \neq 1$ when $t > 0$ and $c > 0$ then the only feasible solution will be

$$e^{-ct_1} = \frac{A(t_2) + [A(t_2) - 2A(t_1)]}{2A(t_1)} = \frac{A(t_2) - A(t_1)}{A(t_1)} = \frac{A(2t_1) - A(t_1)}{A(t_1)}$$

Hence

$$c = -\frac{1}{t_1}\ln\left[\frac{A(t_2) - A(t_1)}{A(t_1)}\right] = -\frac{1}{t_1}\ln\left[\frac{A(2t_1) - A(t_1)}{A(t_1)}\right] \tag{6}$$

We can then solve a by substituting e^{-ct_1} into Equation (5). Thus,

$$\begin{aligned} a &= \frac{A(t_1)}{M(1-e^{-ct_1})} = \frac{A(t_1)}{M\left(1-\frac{A(t_2)-A(t_1)}{A(t_1)}\right)} = \frac{A(t_1)^2}{M[2A(t_1)-A(t_2)]} \\ a &= \frac{A(t_1)^2}{M[2A(t_1)-A(t_2)]} = \frac{A(t_1)^2}{M[2A(t_1)-A(2t_1)]} \end{aligned} \tag{7}$$

4.3. Analysis of Constants

4.3.1. Constant of Retention a

For the values of a to hold, a should be a positive fraction or at most 1, that is $0 < a \leq 1$. Hence $0 < \dfrac{A(t_1)}{M(1-e^{-ct_1})} \leq 1$. Therefore,

$$\begin{aligned} A(t_1) &\leq M\left(1-e^{-ct_1}\right) \\ A(t_1) &\leq M\left(1-\frac{A(t_2)-A(t_1)}{A(t_1)}\right) \\ [A(t_1)]^2 &\leq M[2A(t_1)-A(t_2)] \\ A(t_2) = A(2t_1) &\leq A(t_1)\left(2-\frac{A(t_1)}{M}\right) \end{aligned}$$

Hence this makes $A(t_2) = A(2t_1)$ bounded above. Also, as the retention constant gets closer to 1, $A(t) \to M$ as $t \to \infty$ while on the other hand if the retention constant is closer to 0, $A(t) \to A(t_1)$ as $t \to \infty$. This means that good students will have a higher retention constant (say $0.5 < a < 1$) than poor students. A forgetful student (i.e., $a \to 0$) who continues learning will therefore always be learning the amount he/she initially learnt ($A(t_1)$) as he/she cannot remember what he/she studies. This can be seen when the initially-taken values ($A(t_1)$ and $A(t_2)$) are almost the same.

4.3.2. Rate of Absorption c

The rate of absorption is positive, $c > 0$ and this implies

$$\begin{aligned} 0 < e^{-ct} &< 1 \quad \text{for} \quad t > 0 \\ 0 < \frac{A(2t_1)-A(t_1)}{A(t_1)} &< 1 \\ A(t_1) < A(2t_1) &= A(t_2) \end{aligned}$$

Making $A(2t_1)$ bounded below by $A(t_1)$. Hence $A(t_2) = A(2t_1)$ is bounded.

4.3.3. Relationship Between a and c

From Equation (4), we note that $a = \dfrac{k_1}{k_1+k_2}$ and $c = k_1 + k_2$, hence

$$a = \frac{k_1}{c} \tag{8}$$

which implies that the retention constant is inversely related to the absorption rate, $\left(a \propto \dfrac{1}{c}\right)$.

One can increase c only (without increasing a in the short term) by increasing k_2. This will result in the fact that the person is absorbing extra amount of information that will eventually not be retained. As it can be seen from Equation (3), k_2 contributes negatively to the differential equation and hence it is not rational to increase c in the short term.

Therefore, in the short term, for an individual, there is a fixed amount of information that can be retained in the memory and hence when one forces himself to memorize a lot of information, the fraction retained will be reduced.

However, in the long term, one can increase a and c simultaneously by increasing k_1. This can be achieved by changing the learning strategy, learning materials, understanding of concepts, learning environment, study mates which will result in higher absorption rate and a higher retention constant.

5. RESULTS AND DISCUSSION

Table 1
Values of a and c for $M = 450$ and $A(5) = 100$ with Varying Values of $A(10)$

$A(5)$	$A(10)$	a	c	$A(\infty)$
100	177	0.9662	0.0523	435
100	170	0.7407	0.0713	333
100	160	0.5556	0.1022	250
100	150	0.4444	0.1386	200
100	140	0.3704	0.1833	167
100	130	0.3175	0.2408	143
100	120	0.2778	0.3219	125
100	110	0.2469	0.4605	111
100	101	0.2245	0.9210	101

From Table 1, it can be observed that it is better to have a higher retention constant than a higher absorption rate. A higher retention constant will definitely lead close to total amount needed to be memorized in the long run. On the other, a higher absorption rate without a high retention constant will result in few amount memorized.

Figure 3 shows the various curves for various retention constants as they can be seen with the amount memorized at time 10. An individual with higher retention constant will gradually increase the amount memorized each time while the one with lower retention constant will be around the same amount memorized initially as time goes on even though he tries to absorb more.

Table 1 is based on individual memorizations. The first row shows that if a person is able to memorize one hundred words within the first five days and then one hundred and seventy-seven words in the next ten days such a person can memorize a maximum of Four hundred and thirty-five words in the long run. In the second row, if another person is able to memorize one hundred words within the first five days and then one hundred and seventy words in the next ten days such a person can memorize a maximum of three hundred and thirty-three words as time becomes large.

However, a person with a high constant of retention (0.9662) and a high absorption rate (0.9210) will be able to memorize four hundred and thirty words in five days and four hundred and thirty-five words in ten days. On the contrary, a person with low constant of retention (0.2245) and low absorption rate (0.0523) will be able to memorize twenty-three words in five days and forty-one words in ten days. This goes to prove that a person with a high retention constant and absorption rate is able to recall more than all the others. Further, it confirms that it pays to learn constantly over a long period of time.

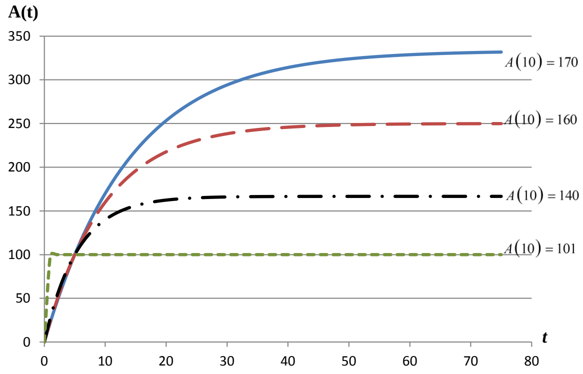

Figure 3
Graph of $A(t)$ Against Time with $M = 450$ and $A(5) = 100$ Against Various $A(10)$

6. CONCLUSION

This study employed the usage of differential equations to model the rate at which students could memorize a given number of axioms and proofs, considering the fact that they will forget some of them along the way. The model brought in its wake the fact that persons who are able to absorb and retain more are able to recollect better than those who can absorb more and retain less. On the other hand, those who can absorb less and retain more have an upper hand in recollection over those who can absorb more and retain less. Consequently it is better to have a higher retention constant than a higher absorption rate. Factors like the learning strategy, learning materials, learning environment, study mates, have either a positive or negative influence on the absorption and retention of the individual in the long term.

7. RECOMMENDATIONS

Based on our study and findings, the following recommendations were made:

- Cardiovascular exercises like walking will help one to think well.
- Reduce stress and get enough rest.
- Right learning strategy should be adopted by employing memorization techniques such as the usage of mnemonics.
- Learning in conducive environments are encouraged for maximum absorption.
- Reduce the intake of memory impeding substances.
- The study should be conducted using actual data compared with calculated data.

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