CSCanada
Studies in Mathematical Sciences
Vol. 4, No. 2, 2012, pp. 25-31
DOI:10.3968/j.sms.1923845220120402.1588

ISSN 1923-8444 [Print]
ISSN 1923-8452 [Online]
www.cscanada.net
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Infinite Series Method for Solving the Improved Modified KdV Equation

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Received 1 January, 2012; accepted 14 May, 2012

Abstract
Infinite Series method obtains some exact solution of non-integrable equations as well as integrable ones. This article is concerned with Infinite Series method for solving the solution of the Improved Modified KdV Equation. It is worth mentioning that this method is based on the idea of the infinite series method. To recapitulate, this investigation has resulted several types of exact solutions of the given (1+1)-dimensional equation that are exact soliton solutions, In addition, some figures of partial solutions are provided for direct-viewing analysis. The method can also be extended to other types of nonlinear evolution equations in mathematical physics.

Key words: Infinite series method; Exact solution; I M KdV equation

A. Asaraai (2012). Infinite Series Method for Solving the Improved Modified KdV Equation. *Studies in Mathematical Sciences, 4*(2), 25-31. Available from URL http://www.cscanada.net/index.php/sms/article/view/j.sms.1923845220120402.1588
DOI: http://dx.doi.org/10.3968/j.sms.1923845220120402.1588

1. INTRODUCTION

The soliton itself is a dramatic new concept in nonlinear science. Here at last, on the classical level, is the entity that field theorists had been postulating for years, a local travelling wave pulse, a lump-like, coherent structure, the solution of a field equation with remarkable stability and particle-like properties. It is intrinsically nonlinear and owes its existence to a balance of two forces; one is linear and acts to disperse the pulse, the other is nonlinear and acts to focus it. Before the soliton, physicists had often talked about wave packets and photons, which are solutions of the linear time- dependent Schrodinger equation. But such packets would always disperse on a time scale inversely proportional to the square of the spread of the packet in wave number space. Nonlinearity is essential for stopping and balancing the dispersion process. Moreover, since the time when the soliton concept was first introduced by Zabusky in 1965[1], As is known, to search for the solitary wave solutions of a nonlinear physical model, we can apply different approaches.

In various fields of science and engineering, many problems can be described by non-linear partial differential (PDEs). It is a universal equation of nonlinear physics and occurs in a huge variety of situations: in nonlinear optics [2], in deep water wave theory [3], in the description of energy transport along alpha-helix proteins [4, 5]. Not only is it ubiquitous, but one can easily give the recipe for the circumstances under which it will obtain. Whereas the NLS was the first-born among soliton equations [6], it was the celebrated Korteweg-deVries (KdV) equation.

Other universal equations, also derived by this process and which also admit soliton solutions, are the modified Korteweg-deVries (MKdV) equation, the derivative nonlinear Schrodinger (DNLS) equation, the three wave interaction (TWI) equations, the Boussinesq equation, the Kadomtsev-Petviashvili (the two-dimensional KdV or KP) equation, the Benjamin-Ono (BO) equation, the intermediate long wave (ILW) equation, the Benney-Roskes-Davey-Stewartson (the two-dimensional NLS) equation, the sine- and sinh-Gordon equations, the massive Thirring model, the Landau-Lifshitz equation, the Gross-Neveu, the Vaks-Larkin-Nambu-Jona Lasiniochiral field models.

The investigation of exact solutions to nonlinear evolution has become an interesting subject in nonlinear science field. One of the most efficient methods of finding soliton excitations for a physical model is the so-called Infinite Series method, the study of the solutions of Partial Differential Equations (PDEs) has enjoyed an intense period of activity over the last forty years from both theoretical and numerical points of view. One of the well-known techniques used to seek analytical–algebraic solutions of differential equations is the Infinite Series method. It has been demonstrated that the Infinite Series method, with the help of symbolic computation, provides a powerful mathematical tool for solving high-dimensional nonlinear evolutions in mathematical physics. The technique we used in this paper is due to Hereman et al. [35]. Here the solutions are developed as series in real exponential functions which physically corresponds to mixing of elementary solutions of the linear part due to nonlinearity. The method of Hereman et al. in 1986 [7] falls into the category of direct solution methods for nonlinear partial differential equations.

In recent years, other methods have been developed, such as the Backlund transformation method[8], Darboux transformation [9], tanh method[10, 11], extended tanh function method[12], modified extended tanh function method [13], the generalized hyperbolic function[14, 15], the variable separation method[16], First Integral method[17- 19] and f-expansion method[20].

The aim of this paper is organized as follows: In Section 1,at first, we briefly give the steps of Infinite Series method. In Section 2, by using the results obtained in Section 1, and apply the method to solve the Improved Modified KdV equation, lastly some figures of partial solutions are provided for direct-viewing analysis.

2. INFINITE SERIES METHOD

Consider a general nonlinear partial differential equation in the form:

$$F(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx} \dots) = 0, \tag{1}$$

Where is the solution of nonlinear PDE equation (1). Furthermore, the transformations which are used are as follows:

$$u(x,t) = g(\xi), \qquad \xi = l_1(x - l_2 t). \tag{2}$$

Where is constant. Using the chain rule, it can be found that

$$\frac{\partial}{\partial t}(\cdot) = -l_1 l_2 \frac{\partial}{\partial \xi}(\cdot), \quad \frac{\partial}{\partial x}(\cdot) = l \frac{\partial}{\partial \xi}(\cdot), \quad \frac{\partial^2}{\partial x^2}(\cdot) = l_1^2 \frac{\partial^2}{\partial \xi^2}(\cdot), \dots \tag{3}$$

At present, equation (3) is employed to change the nonlinear PDE equation (1) to nonlinear ordinary differential equation

$$G(h(\xi), h_\xi(\xi), h_{\xi\xi}(\xi), \dots) = 0 \tag{4}$$

Next, we apply the approach of Hereman et al. [(1986)]. We solve the linear terms and then suppose the solution in the form

$$h\ \xi = \sum_{n=1}^{\infty} a_n f^n(\xi). \tag{5}$$

Where $h(\xi)$ is a solution of linear terms and the expansion coefficients a_n(n=1,2,3,...) are to be determined. To deal with the nonlinear terms, we need to apply the extension of Cauchy's product rule for multiple series.

Lemma 1. (Extension of Cauchy's product rule). Have

$$\prod_{j=1}^{I} F^{(j)} = \sum_{n=1}^{\infty} \sum_{r=I-1}^{n-1} \sum_{p=I-2}^{q-1} \cdots \sum_{m=2}^{s-1} \sum_{l=1}^{m-1} a_l^{(1)} a_{m-l}^{(2)} \cdots a_{r-p}^{(I-1)} a_{n-r}^{(I)}, \tag{6}$$

That,

$$F^{\ j} = \sum_{\vartheta=1}^{I} a_{\vartheta}^{(j)}.$$

represents infinite convergent series, for proof seen [7].

Substituting (5) into (4) yields recursion relation which gives the values of the coefficients.

3. APPLICATION

In this section, it is aimed to discuss the Improved Modified KdV equation, written in the form of the following equations:

$$u_t + u^2 u_x + u_{xxx} - u_{xxt} = 0 \tag{7}$$

The celebrated Improved Modified KdV (IMkdV) equation can be solved by many methods, e.g. A trial function Method [21], Modified extended direct algebric Method [22], First Integral method [23]. In this paper, we will apply He`s Infinite series method to search for its solitary solutions. It is necessary to state that equation (7) plays an important role in nonlinear physics.

By using the transformation

$$u(x,t) = U(\xi) \qquad \xi = \alpha(x - \beta\ t) \tag{8}$$

Equation (7) changes into:

$$\alpha^2(1+\beta)U_{\xi\xi\xi} - \beta U_{\xi}(\xi) + U^2(\xi)U_{\xi}(\xi) = 0 \tag{9}$$

Where by integrating the Eq. (9), respect to , it can be found that

$$\alpha^2(1+\beta)U_{\xi\,\xi} - \beta\ U(\xi) + \frac{1}{3}U^3(\xi) = 0 \tag{10}$$

According to the infinite series method, the linear Eq. (10) has the solution in the form

$$U(\xi) = exp\left(\sqrt{\frac{\beta}{\alpha^2(1+\beta)}}\right)\xi.$$

Consequently, with looking for the solution of Eq. (10), the following solutions will be attained:

$$V\ \xi\ = \sum_{1}^{\infty} a_n\, exp(n\sqrt{\frac{\beta}{\alpha^2\ (1+\beta)}}\ \xi\ ). \qquad (11)$$

Substituting (11) into (10) and by using Lemma 1, we obtain the recursion relation follows
a_1 is arbitrary constant,

a_2=0

$$a_3 = -\frac{1}{8}\frac{1}{3\beta}\sum_{m=2}^{3-1}\sum_{l=1}^{m-1} a_l a_{m-l} a_{3-m} = -\frac{1}{2^3}\frac{1}{3\ \beta}a_1^3,$$

a_4=0

$$a_5 = -\frac{1}{2\ 4}\frac{1}{3\beta}\sum_{m=2}^{5-1}\sum_{l=1}^{m-1} a_l a_{m-l} a_{5-m} = \frac{1}{2^6}\left(\frac{1}{3\ \beta}\right)^2 a_1^5,$$

a_6=0

$$a_n = -\frac{1}{n^2-1}\frac{1}{3\beta}\sum_{m=2}^{n-1}\sum_{l=1}^{m-1} a_l a_{m-l} a_{n-m}\ , \qquad \mathbf{n\geq 3.} \qquad (12)$$

By assuming, a_1 is arbitrary and a_2 in the recursion relation (12), it can be found that

$$a_n = \begin{cases} a_{2\ k} = 0, \\ a_{2\ k+1} = (-1)^k \left(\frac{1}{3\ \beta}\right)^k \frac{a_1^{2\ k+1}}{2^{3\ k}}, \end{cases} \qquad k=1,2,3,... \qquad (13)$$

Substituting (13) into (11) gives

$$U(\xi) = \sum_{k=0}^{\infty}(-1)^{3k}\left(\frac{1}{3\beta}\right)^k \frac{a_1^{2k+1}}{2^{3k}} exp((2k+1)\sqrt{\frac{\beta}{\alpha^2(1+\beta)}}\ \xi) \qquad (14)$$

Hence,

$$U(\xi) = \frac{a_1\, exp(\sqrt{\frac{\beta}{\alpha^2(1+\beta)}}\ \xi)}{1+\frac{1}{3\ \beta}\frac{a_1^2}{8}\, exp(2\sqrt{\frac{\beta}{\alpha^2(1+\beta)}}\ \xi)}, \qquad (15)$$

If we choose and substituting this condition into (15), we find

$$U(\xi) = \pm \frac{2\sqrt{6\,\beta}exp(\sqrt{\frac{\beta}{\alpha^2(1+\beta)}}\ \xi)}{1 + exp\ (2\ \sqrt{\frac{\beta}{(\alpha^2\ \ 1+\beta)}}\ \xi)} \qquad (16)$$

Case I:

In Eq.(16), if β<0, in view of the fact that $sec\ t = \frac{2e^{t}}{1+e^{2t}}$ in (16), the exact solution to equation (10) will be found as follows:

$$U(\xi) = \pm\sqrt{6\ \beta}sech(\ \sqrt{\frac{\beta}{\alpha^2\ (1+\beta)}}\ \xi) \qquad (17)$$

Case II:

If β<0, since $sec\ t = \frac{2e^{it}}{1+e^{2it}}$ in (16), the exact solution to equation (10) will be created as follows:

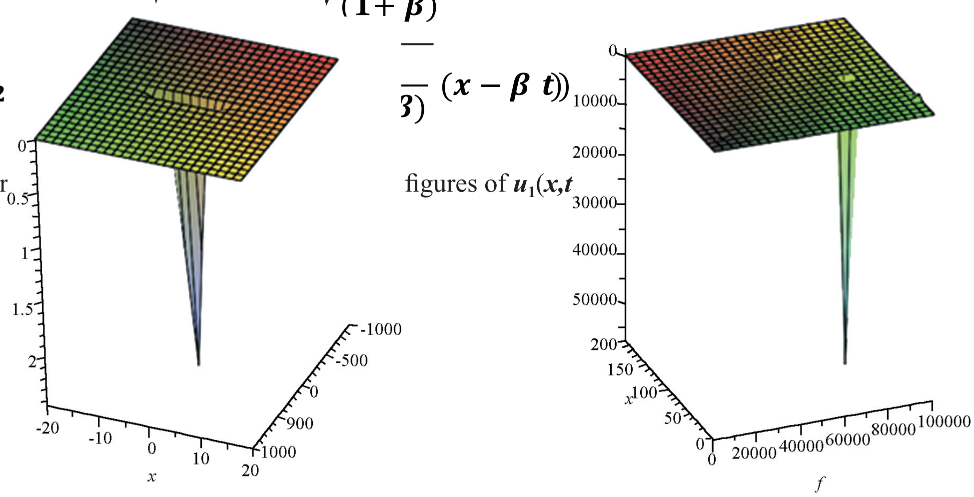
$$U(\xi) = \pm i\sqrt{6\,\beta}sec(\sqrt{\frac{\beta}{\alpha^2\,(1+\beta)}}\ \xi) \qquad (18)$$

Therefore, by inspection of conditions Case I and Case II in above and by means of Eq. (8), the exact soliton solutions of the Improved Modified KdV (IMkdV) equation (7) will be found as follows:

$$u_1\,(x,t) = \pm\sqrt{6\ \beta}\,sech(\sqrt{\frac{\beta}{(1+\beta)}}\ (x-\beta\ t)). \qquad (19)$$

$$u_2 \ \ldots \ \beta)\ (x-\beta\ t)) \qquad (24)$$

For dir ... figures of $u_1(x,t$



(a) Graphic of the Soliton Solution $u_1(x,t)$ are Shown at "-".

(b) Graphic of the SolitonSolution $u_2(x,t)$ are Shown at "+".

Figure 1
As a Final Notion, These Solutions Are Considered as New Exact Soliton Solutions for the Improved Modified Equation

4. CONCLUSION

In this study, Infinite series method was described to find exact solutions of the Improved Modified KdV (IMkdV) equation. Consequently, four exact soliton solutions were obtained to the IMkdV equation. In spite of the fact that these new soliton solutions may be important for physical problems, this study also suggests that one may find different solutions by choosing different methods. Therefore, this method can be utilized to solve many equations of nonlinear partial differential equation arising in the theory of soliton and other related areas of research.

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