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Statistical Appraisal of Maximum Age Requirement for Commercial Airplanes in Nigeria

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Abstract: This paper proposes and uses a factor of relative age difference for each plane termed relative "plane age", index. Using these indexes and their ranks, it is shown that the enunciated mandatory upper age limit of 20 years is approximately the mean age of the commercial planes in the country estimated to be 20.7 years, but higher than the median age of the planes found to be 19.4years. Thus if median age of 19.4 or about 19 years rather than 20 years is to be set as the required upper age limit, then only about 33 or 34 rather than 37 commercial planes would be properly eligible to fly Nigeria's airspace. Statistically significant differences in age are found to exist between commercial planes that may importantly affect their operation. Relative "plane age" indexes that are positive with a value of 17 or larger so that the corresponding planes are younger than at least 42 and older than at most 25 other planes and aged at most 15.3 years are statistically significant; while those relative "plane age" indexes that are negative with a value of at most 20 so that the corresponding planes are younger than at most 23 and older than at least 43 other planes and aged at least 21.2 years are statistically significant. Hence if age is to be considered as a statistical factor affecting air-worthiness of commercial planes, then the upper age limit of 15.3 or 15 years should be preferred and used as a selection eligibility criterion for commercial planes in Nigeria. This will in effect imply that no plane aged above 15.3 years may be allowed to fly resulting in only about

26 commercial planes rather than 37 as is the case under the current dispensation being able to properly and normally use Nigeria's airspace.

Key words: Rank-order; Relative plane age; Index; Chi-square; Sample estimate; Relative performance

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1. INTRODUCTION

The Nigerian Airspace Management Agency (NAMA) following a spate of plane crashes in Nigeria and particularly after the 2012 crash of a plane operated by Dana-Air, has now mandatory regulated that no commercial airplane using Nigeria's airspace should be older than twenty years of age, otherwise such airplane cannot normally operate in the country.

There have however been arguments on the merits and demerits of this requirement. Some argue that age is not an important factor affecting safety of aircrafts but the regularity of aircrafts maintenance. Others on the other hand argue that the serviceability of aircrafts like other mechanical and electronic systems is a function of age which would ultimately result in system failure occasioned by the expiration of the systems maximum natural life span notwithstanding the regularity of maintenance.

We will however for the purpose of the present paper only assume that age is an important factor affecting aircraft safety even if perhaps not in the absolute. The intention here is to statistically rank-order the commercial airplanes currently using Nigeria's airspace on the basis of their age and hence presumed safety-level and thereby also enable informed determination of the likely number of commercial aircrafts that are qualified to use Nigeria's airspace under the twenty years maximum age requirement. Other researchers that have worked in this area or something similar include Raivo, Sven, Priit, and Jaak (2012), Adler and Golany (2001), Charnes, Clark, Cooper, and Golany (1985a), Dijk, Fok, and Paap (2012), Schwarz andWyer Jr (1985), Drew (2012), Groeneveld (1990), Kennedy (2012, June), Larichev and Moshkovich (1995), Uwadiae (2000), Mantel (1966), Allen and Sharpe (2005), Nissan (1994), Omoleke (2012), and Yu (2000).

2. THE PROPOSED METHOD

Oyeka, Ebuh, and Michael (2012) have developed a statistical method for preferentially rank-ordering subjects, objects or entities by level of performance in a contest, test or any set objective. For example often assessors, decision makers, judges, teachers etc, may assess, examine or judge a sample from a population of subjects and score them for employment, placement in educational institutions or for selection to fill vacant positions when opportunities are limited. A medical or health researcher or health management official may have data or information on subjects or patients on their state of health, medical test results, level of concentration of some contaminants, disease load, injury levels and other such conditions, and may wish to relatively rank-order the subjects by the severity of their condition to guide decisions on the distribution and use of amenities when supplies are limited. In business, commerce, industry and governmental affairs, one may wish to know how various outfits, producers, suppliers and distributors of goods and services such as banks, transport operators ministries, parastatals etc compare in performance when juxta-posed against one another to guide any interventionist remedial actions by management or supervisory body. The problem often before decision makers is how using these observations to rationally select the required number of subjects or options to ensure that meritocracy are upheld in the presence of scarcity.

Oyeka, Ebuh, and Michael (2012) in their paper tried to address this problem and developed an index for systematically rank-ordering subjects, objects or items in these and similar situations according to level of performance or achievement in the process. To do this, the authors defined the count variable as.

$$\mu_{ij} = \begin{cases} 1, & if \quad x_i > x_j, \quad (or \quad x_j < x_i) \\ 0, & if \quad x_i = x_j \\ -1, & if \quad x_i < x_j, \quad (or \quad x_j > x_i) \end{cases} \tag{1}$$

For i, j, = 1, 2,…, n, i ≠ j, where x_i is the score or observation on the ith subject randomly drawn from the population of subjects exposed to some trial or experiment for i =1, 2, …,n.

Note that depending on the problem of interest, the values of u_{ij} may be viewed and interpreted as performance units (scores)” or “time-space displacement performance units. Thus the values 1, 0 and −1 may be interpreted as respectively representing positive, zero and negative” performance units or indicators.

Now let

$$\pi^{+}_{i} = P(\mu_{ij} = 1) \; ; \pi^{0}_{i} = P(\mu_{ij} = 0) \; ; \pi^{-}_{i} = P(\mu_{ij} = -1) \tag{2}$$

for i = 1, 2, …, n, i≠ j, where

$$\pi^{+}_{i} + \pi^{0}_{i} + \pi^{-}_{i} = 1 \tag{3}$$

Also define

$$W_i = \sum_{j=1}^{n} u_{ij} \tag{4}$$

Note that π^{+}_{i}, π^{0}_{i}, and π^{-}_{i} are respectively the proportions of all the subjects in the population whose scores in the test are lower than, equal to or higher than the score earned by the ith subject for i = 1, 2,…,n.

The sample estimates of these probabilities or proportions are shown to be respectively:

$$\hat{\pi}^{+}_{i}=\frac{f_i^{+}}{n-1};\hat{\pi}^{0}_{i}=\frac{f_i^{0}}{n-1};\hat{\pi}^{-}_{i}=\frac{f_i^{-}}{n-1} \tag{5}$$

where f_i^{+} , f_i^{0} and f_i^{-} are respectively, the number of 1s, 0s and -1s in the frequency distribution of the n–1 values of these numbers in u_{ij}; i,j = 1,2,…,n; i≠j. That is f_i^{+} , f_i^{0} and f_i^{-} are respectively the numbers of subjects in the sample whose scores in the test are lower than, equal to or higher than the score by the ith subject, i = 1, 2,…,n.

The authors under reference also showed that the sample estimate of W_i , and the corresponding sample variance are respectively:

$$W_i=(n-1)\left(\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}\right)=f_i^{+}-f_i^{-} \tag{6}$$

and $$Var(W_i)=(n-1)\left(\hat{\pi}^{+}_{i}+\hat{\pi}^{-}_{i}-\left(\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}\right)^2\right) \tag{7}$$

Note that $$\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}=\frac{W_i}{n-1}=\frac{f_i^{+}-f_i^{-}}{n-1} \tag{8}$$

provides a sample estimate of the proportion of all subjects in the population whose scores in the test are exceeded by the score of the ith subject and hence is a sample estimate of the gap on relative performance or response by the ith subject in the test when compared with all other subjects in the population. The authors used $\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}$ as a measure or index of relative performance by the ith subject, object or item from the sampled population in comparison with all other subjects in the population and obtained the sample estimate of its variance as

$$Var\left(\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}\right)=\frac{Var(W_i)}{(n-1)^2}=\frac{\hat{\pi}^{+}_{i}+\hat{\pi}^{-}_{i}-\left(\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}\right)^2}{n-1} \tag{9}$$

for i = 1,2,...,n.

The authors further developed a test statistic for testing the null hypotheses that the ith subject neither performs better nor worse than other subjects in the population of subjects exposed to the test as

$$\chi^2=\frac{W_i^{2}}{Var(w_i)}=\frac{W_i^2}{(n-1)\left(\hat{\pi}^{+}_{i}+\hat{\pi}^{-}_{i}-\left(\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}\right)^2\right)}=\frac{(n-1)\left(\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}\right)^2}{\left(\hat{\pi}^{+}_{i}+\hat{\pi}^{-}_{i}-\left(\hat{\pi}^{+}_{i}-\hat{\pi}^{-}_{i}\right)^2\right)} \tag{10}$$

A test statistic is also developed by Oyeka, Ebuh, and Michael (2012) for testing the null hypothesis of the existence of no differential in relative performance or scores between any two subjects, objects or items i and k from the sampled population which is

$$\chi^2_{ik} = \frac{\left(W_i - W_k\right)^2}{Var\left(w_i\right) + Var\left(W_k\right)} = \frac{(n-1)\left(\left(\hat{\pi}^+_i - \hat{\pi}^-_i\right) - \left(\hat{\pi}^+_k - \hat{\pi}^-_k\right)\right)^2}{\left(\hat{\pi}^+_i + \hat{\pi}^-_i - \left(\hat{\pi}^+_i - \hat{\pi}^-_i\right)^2\right) + \left(\hat{\pi}^+_k + \hat{\pi}^-_k - \left(\hat{\pi}^+_k - \hat{\pi}^-_k\right)^2\right)} \qquad (11)$$

Eqns 10 and 11 each has approximately the chi-square distribution with 1 degree of freedom for a sufficiently large n and i,k = 1,2,…,n; i≠k. The null hypotheses are each rejected if the calculated chi-square values are greater than the tabulated critical chi-square value at a specified *a* level otherwise H_0 is accepted.

To avoid a situation in which the denominators of Eqns10 and 11 are zero because the ith subjects test score or response is greater (or less) than those of all other subjects, objects or items in the sampled population, that is in which f_i^+=n−1 and f_i^-=0 or vise versa so that $\hat{\pi}^+_i$= 1.0 and $\hat{\pi}^-_i$= 0.0 or vise versa yielding a meaningless value of the test statistic, the authors under reference recommended that in such a case a correction factor of $\frac{1}{2(n-1)}$ be subtracted from $\hat{\pi}^+_i$and added to $\hat{\pi}^-_i$ or vise-versa depending on which of the two currently has a value of 1 or a value of "O" for that subject before calculating the variance of W_i , i = 1,2,…,n.

As noted above the index W_i– the difference between number of subjects whose scores are lower and the number of subjects whose scores are higher than the score by the ith subject; or equivalently $\hat{\pi}^+_i - \hat{\pi}^-_i$.

These procedures are adopted here; the estimated gap in relative performance or response by the ith subject over and above all other subjects in the sample may be used to preferentially rank-order or rate the subjects, items or entities by their performance or response in the test or condition. This index is here used to determine the relative gaps in the ages of commercial aircrafts currently operating in Nigeria and hence assess the relative quality of these planes if based only on their ages.

To do this we would rank-orderW_i or $\hat{\pi}^+_i - \hat{\pi}^-_i$, for i = 1,2,…,n, by their magnitudes other from the largest (highest) to the smallest (lowest) or from the smallest(lowest) to the largest(highest) assigning the largest value the rank of "1"(or n), the next largest the rank of 2(or n – 1) and so on, until the smallest value is assigned the lowest rank n(or 1). All tied values of W_i are as usual assigned their mean ranks. This procedure provides a preferential ordering of the subjects by their assigned ranks r_i which may now be used as a preferential ranking index to rank-order the subjects or items on the condition of interest for preferential selection and decision purposes as may be desired, in this particular case for the analysis of our data on the ages of aircrafts operating in Nigeria.

Finally, however, before the application proper, it would be instructive to note that the above ranking procedure yields essentially the same rank for each subject as would have been obtained if only subjects" scores had been ranked. Nevertheless the procedure enables the researcher immediately have a birds eye-view in the form of a spread sheet of the overall ranking of subjects relative to one another in a performance or response test and also determine by how much a given subject fares better averagely as well as or worse than other subjects in the population which provides additional useful information. Based on these rankings, the policy implementer may decide to introduce any desired interventionist measures for subjects either right of the average or center, left of the centre or both depending on the condition being remediated.

The method also enables easy and quick estimation with minimal calculations of the percentiles and other tiles of the distribution of the population of interest using their ranks. Thus the k^th^ percentile of the distribution is the value of the observation x_i corresponding to $W_i=n-1$

$$(\hat{\pi}^+_i - \hat{\pi}^-_i) \text{ with rank. } r_{(i)} = \begin{cases} k((n+1)/p) & if \quad n \quad is \quad odd \\ \dfrac{k(n/p)+k(n/p)+1}{2}, & if \quad n \quad is \quad even \end{cases} \qquad (12)$$

for k= 1,2,3,…,99; and P = 100; i = 1,2,…,n.

3. APPLICATION AND RESULT

Table 1 presents data on the ages of commercial airplanes flying Nigeria's airspace. There are 10 known airlines in Nigeria with a total of 96 planes 68 of which have their ages stated (Tolex, 2012). For simplicity and brevity only code names are used to designate the airlines with the planes owned by each air line assigned serial numbers. Thus if Air Transa is an airline, in Nigeria with Y planes say, then this airline would have the code name T.A and its plane numbers 1 and 5 say would be designated as T.A.I and T.A.5 respectively. The resulting list of the 68 planes with information on their ages in years is presented in Table 1 as shown below.

Table 1
Ages (in years) of 68 Commercial Air Planes in Nigeria

Plane No	Age	Plane No	Age	Plane No	Age	Plane No	Age	Plane No	Age
CA.1	22.2	NA.4	12.7	AA.9	11.2	AD.1	20.9	AK.2	41.3
CA.2	21.7	NA.5	12.6	AA.10	11.1	AD.2	21.2	AK.3	25.9
CA.3	12.9	NA.6	15.2	AA.11	3.4	AD.3	21.7	AK.4	38.1
CA.4	19.4	NA.7	15.3	AA.12	3.3	AD.4	21.6	AK.5	41.3
CA.5	19.0	NA.8	15.2	AA.13	2.8	AD. Mean Age	21.4	AK. Mean Age	34.5
CA.6	19.4	NA.9	18.2	AA.14	2.7	NF.1	19.3	AM.1	25.7
CA.7	19.9	NA.10	6.7	AA.15	4.0	NF.2	18.2	AM.2	24.5
CA.8	19.9	NA Mean Age	13.5	AA.16	3.9	NF.3	17.7	AM.3	24.6
CA.9	20.5	AA.1	5.4	AA. Mean Age	5.5	NF. Mean Age	18.4	Am.4	21.1
CA.10	20.7	AA.2	5.2	AC9.1	30.2	AI.1	22.3	AM.5	24.4
CA.11	21.0	AA.3	5.2	AC9.2	29.0	AI.2	18.4	AM. Mean Age	24.1
CA.12	20.2	AA.4	4.7	AC9.3	29.0	AI.3	22.3	AO.1	25.6
CA.Mean Age	19.7	AA.5	4.6	AC9.4	22.9	AI.4	22.5	AO.2	18.7
NA.1	13.0	AA.6	4.2	AC9.5	21.3	AI.5	22.4	AO. Mean Age	22.2
NA.2	13.0	AA.7	4.2	AC9.6	21.2	AI. Mean Age	21.6		
NA.3	12.8	AA.8	11.3	AC9. Mean Age	25.6	Ak.1	26.1		

Source: Tolex (2012).

It can easily be calculated from Table 1 that the mean age of these planes is about 20.7years. Altogether 34 of the planes are at most 19.4 years old, the median age of these planes while 34 planes are this age or older. These information and more may however be readily obtained using the method presented above. Thus application of Eqn 1 to the data of Table 1 helped us to obtain the values of u_{ij}, i,j=1,2,…,n = 68; i ≠ j. Then using these values with the other Equations as presented above enabled us to obtain sample estimates of $f_i^+, f_i^0, f_i^-, \hat{\pi}^+_i, \hat{\pi}^0_i, \hat{\pi}^-_i$, W_i, r_i, the rank assigned to W_i and other statistics which are presented in Table 2 for i = 1,2,…,68 as shown below.

Table 2
Quality Rating of Commercial Air Planes in Nigeria

S/N	Carrier No.	Identification Age	f_i^+	f_i^0	f_i^-	$\hat{\Pi}_i^+$	$\hat{\Pi}_i^0$	$\hat{\Pi}_i^-$	W_i	VarW_i	χ_1^2	P-value	Preferential rank
1.	CA.1	22.2	18	0	49	0.269	0.000	0.731	-31	52.657	18.250	0.0000	50
2.	CA.2	21.7	19	1	47	0.284	0.015	0.701	-28	54.294	14.440	0.0000	48.5
3.	CA.3	12.9	47	0	20	0.701	0.000	0.299	27	56.119	12.816	0.0000	21
4.	CA.4	19.4	33	1	33	0.493	0.015	0.493	0	65.995	0.000	1.000	34.5
5.	CA.5	19.0	36	0	31	0.537	0.000	0.463	5	66.074	0.378	P>0.05	32
6.	CA.6	19.4	33	1	33	0.493	0.015	0.493	0	65.995	0.000	1.000	34.5
7.	CA.7	19.9	31	1	35	0.463	0.015	0.522	-4	65.756	0.243	P>0.05	36.5
8.	CA.8	19.9	31	1	35	0.463	0.015	0.522	-4	65.756	0.243	P>0.05	36.5
9.	CA.9	20.5	29	0	38	0.433	0.000	0.567	-9	65.791	1.231	0.0869	39
10.	CA.10	20.7	28	0	39	0.418	0.000	0.582	-11	65.194	1.856	0.0869	40
11.	CA.11	21.0	26	0	41	0.388	0.000	0.612	-15	63.652	3.535	0.0635	41.5
12.	CA.12	20.2	30	0	37	0.448	0.000	0.522	-7	66.269	0.739	P>0.05	38
13.	NA.1	13.0	45	1	21	0.672	0.015	0.313	24	57.396	10.036	0.0000	22.5
14.	NA.2	13.0	45	1	21	0.672	0.015	0.313	24	57.396	10.036	0.0000	22.5
15.	NA.3	12.8	48	0	19	0.716	0.000	0.284	29	54.448	15.446	0.0000	20
16.	NA.4	12.7	49	0	18	0.731	0.000	0.269	31	52.657	18.250	0.0000	19
17.	NA.5	12.6	50	0	17	0.746	0.000	0.254	33	50.746	21.460	0.0000	18
18.	NA.6	15.2	43	1	23	0.642	0.015	0.343	20	60.032	6.663	0.0000	24.5
19.	NA.7	15.3	42	0	25	0.627	0.000	0.373	17	62.687	4.610	0.0389	26
20.	NA.8	15.2	43	1	23	0.642	0.015	0.343	20	60.032	6.663	0.0000	24.5
21.	NA.9	18.2	41	0	26	0.612	0.000	0.388	15	63.65	3.535	0.0635	27.5
22.	NA.10	6.7	54	0	13	0.806	0.000	0.194	41	41.910	40.109	0.0000	14
23.	AA.1	5.4	55	0	12	0.821	0.000	0.179	43	38.398	48.154	0.0000	13
24.	AA.2	5.2	56	1	10	0.836	0.015	0.149	46	34.413	31.582	0.0000	11.5
25.	AA.3	5.2	56	1	10	0.813	0.015	0.149	46	34.413	31.582	0.0000	11.5
26.	AA.4	4.7	58	0	9	0.866	0.000	0.134	49	30.159	79.611	0.0000	10
27.	AA.5	4.6	59	0	8	0.881	0.000	0.119	51	27.179	95.699	0.0000	9
28.	AA.6	4.2	60	1	6	0.896	0.015	0.090	54	22.443	129.929	0.0000	7.5
29.	AA.7	4.2	60	1	6	0.896	0.015	0.090	54	22.443	129.929	0.0000	7.5
30.	AA.8	11.3	51	0	16	0.761	0.000	0.239	35	48.716	25.146	0.0000	17
31.	AA.9	11.2	52	0	15	0.776	0.000	0.224	37	20.433	66.999	0.0000	16
32.	AA.10	11.1	53	0	14	0.791	0.000	0.209	39	44.299	34.335	0.0000	15
33.	AA.11	3.4	64	0	3	0.955	0.000	0.045	61	11.463	324.610	0.0000	4
34.	AA.12	3.3	65	0	2	0.970	0.000	0.030	63	7.761	511.403	0.0000	3
35.	AA.13	2.8	66	0	1	0.985	0.000	0.015	65	3.940	1072.335	0.0000	2
36.	AA.14	2.7	67	0	0	1.00	0.000	0.000	67	1.876	2392.857	0.0000	1
37.	AA.15	4.0	62	0	5	0.925	0.000	0.075	57	18.507	175.535	0.0000	6
38.	AA.16	3.9	63	0	4	0.940	0.000	0.060	59	15.045	231.373	0.0000	5

S/N	Carrier No.	Identification Age	f_i^+	f_i^0	f_i^-	$\hat{\Pi}_i^+$	$\hat{\Pi}_i^0$	$\hat{\Pi}_i^-$	W_i	$VarW_i$	χ_1^2	P-value	Preferential rank
39.	AC9.1	30.2	3	0	64	0.045	0.000	0.955	-61	12.064	308.438	0.0000	65.5
40.	AC9.2	29.0	4	1	62	0.060	0.015	0.925	-58	15.786	213.181	0.0000	63.5
41.	AC9.3	29.0	4	1	62	0.060	0.015	0.925	-58	15.786	213.181	0.0000	63.5
42.	AC9.4	22.9	13	0	54	0.194	0.000	0.806	-41	41.910	40.109	0.0000	55.5
43.	AC9.5	21.3	22	0	45	0.328	0.000	0.672	-25	59.104	8.945	0.0000	45.5
44.	AC9.6	21.2	23	1	43	0.343	0.015	0.642	-20	60.025	6.664	0.0000	43.5
45.	AD.1	20.9	26	0	41	0.388	0.000	0.612	-15	63.641	3.535	0.0000	41.5
46.	AD.2	21.2	23	1	43	0.343	0.015	0.642	-20	60.024	6.664	0.0000	43.5
47.	AD.3	21.7	19	1	47	0.284	0.015	0.701	-28	54.294	14.440	0.0000	48.5
48.	AD.4	21.6	20	1	46	0.299	0.015	0.687	-26	10.089	67.004	0.0000	47
49.	ANF.1	19.3	35	0	32	0.522	0.000	0.478	3	66.866	0.135	0.0000	33
50.	ANF.2	18.2	39	1	27	0.582	0.015	0.403	12	63.846	2.255	0.0000	29
51.	ANF.3	17.7	41	0	26	0.612	0.000	0.388	15	63.635	3.535	0.0000	27.5
52.	AI.1	22.3	17	1	49	0.254	0.015	0.731	-32	50.711	20.193	0.0000	51.5
53.	AI.2	18.4	38	0	29	0.567	0.000	0.433	9	65.791	1.231	0.0000	30.5
54.	AI.3	22.3	17	1	49	0.254	0.015	0.731	-32	50.711	20.193	0.0000	51.5
55.	AI.4	22.5	15	0	52	0.224	0.000	0.776	-37	46.567	29.398	0.0000	53
56.	AI.5	22.4	14	0	53	0.209	0.000	0.791	-39	44.299	34.335	0.0000	54
57.	AK.1	26.1	6	0	61	0.090	0.000	0.910	-55	21.850	138.439	0.0000	62
58.	AK.2	41.3	0	1	66	0.000	0.015	0.985	-66	1.005	4334.325	0.0000	67.5
59.	AK.3	35.9	7	0	60	0.104	0.000	0.896	-53	25.075	112.026	0.0000	61
60.	AK.4	38.1	3	0	64	0.045	0.000	0.955	-61	11.463	324.618	0.0000	65.5
61.	AK.5	41.3	0	1	66	0.000	0.015	0.985	-66	1.005	4334.325	0.0000	67.5
62.	AM.1	25.7	8	0	59	0.119	0.000	0.881	-51	28.179	92.302	0.0000	60
63.	AM.2	24.5	11	0	56	0.164	0.000	0.836	-45	36.776	55.063	0.0000	57
64.	AM.3	24.6	10	0	57	0.149	0.000	0.851	-47	34.029	64.914	0.0000	58
65.	AM.4	21.1	22	0	45	0.328	0.000	0.672	-23	59.104	8.950	0.0000	45.5
66.	AM.5	24.4	13	0	54	0.194	0.000	0.806	-41	41.910	40.109	0.0000	55.5
67.	AO.1	25.6	9	0	58	0.134	0.000	0.866	-49	31.164	77.043	0.0000	59
68.	AO.2	18.7	38	0	29	0.567	0.000	0.433	9	65.791	1.231	0.0000	30.5

The so called gap in relative performance by the ith subject is estimated as

$$W_i = (n-1)\left(\hat{\pi}^+_i - \hat{\pi}^-_i\right) = f_i^+ - f_i^-$$

with rank r_i, when used with data on ages of planes may be viewed and interpreted as a relative plane age index of the ith plane in comparison with all other planes in the sampled population. W_i is the total number of planes in the sampled population the ith plane is younger than, less than the total number of planes the plane is older than i=1,2,…,n.

If the ith plane is younger than all other planes and the Ws are not tied in their values, then $W_i=(n-1)=f_i^+$, $f_i^-=0$; $\hat{\pi}^+_i=1$, $\hat{\pi}^-_i=0$, and the rank $r_i = 1$; so that the ith plane is considered the most preferred in the preferential rank ordering of the planes on the basis of age. If the ith plane is younger than as many planes as it is older than, then $W_i=0$, $f_i^+= f_i^-$, $\hat{\pi}^+_i=\hat{\pi}^-_i$, and r_i is the median rank so that the ith plane would be considered better or more preferred than one-half, and worse or less preferred than another one-half of the planes. If the ith plane is older than all the other planes, then $W_i=-(n-1)=f_i^+$, $f_i^-=0$; $\hat{\pi}^+_i=0$, $\hat{\pi}^-_i=1$, and $r_i=n$, the lowest assigned rank, so that the ith plane would be considered the least preferred among the planes in terms of age.

Thus, the larger and positive the value of W_i is, the more highly rated and preferred is the ith plane relative to other planes in terms of age; the smaller and negative the value of W_i, the lower the rating the ith plane and the more the ith plane is considered less preferred and less air worthy than other planes on the basis of age only.

Hence in terms of commercial planes of interest f_i^+, f_i^0 and f_i^- may be interpreted as respectively the number of times the ith commercial plane is younger as old as, or older than all other planes, so that $W_i=f_i^+-f_i^-$ is the margin or gap, that is the number of planes the ith commercial plane is younger, less than the number of planes it is older than. In other words f_i^+, f_i^0 and f_i^-, may be interpreted as respectively the number of other planes the ith plane is younger as old as, or older than the planes. Hence W_i may be used as a measure of the total number of planes, the ith plane is younger less the number of planes it is older than. The larger and positive the value of W_i is, the younger the ith plane is compared with all other planes and hence the more preferable in terms of age. On the other hand the smaller and negative the value of W_i is, the older the ith plane is compared with other planes and hence the less preferable to other planes in terms of age.

Therefore rank ordering W_i 's from the largest positive value to the smallest negative value would provide a rank-based preferential selection index for the commercial planes based on their ages for i=1,2,...,n, as shown in Table 2. It can be easily seen from the assigned ranks r_i shown in Table 2 that the best rated plane in terms of being the youngest in age is AA.14 aged 2.7 years with $W_i = 67$ and hence ranked number 1, while the lowest rated planes are AK.2 and AK.5 each aged 41.3 years with $W_i = -66$, and hence ranked 67.5 each. Thus AA.14 is younger than all the other 67 commercial air planes while AK.2 and AK.5 are each older than all other commercial airplanes currently operating in Nigeria.

Notice from Table 1 that planes CA.4 and CA.6 with ranks 34.5 and each aged 19.4 years are the planes that occupy the middle most position or the median of the age distribution of the commercial planes. In other words these two planes are younger than one half and older than the other one-half of the airplanes in Nigeria.

This same information and more are however more lucidly conveyed by the results of Table 2 in terms of the f_i's, π_i's, W_i's and r_i's, the ranks assigned to W_i's. Thus it is easily seen from this Table that air planes CA.4 and CA.6 each with $W_i=0$ are each younger than as many planes $f_i^+= 33$) as it is older than ($f_i^-= 33$) also; and each is as old as only one other plane ($f_i^0= 1$). Hence the age 19.4 years of these two planes is the median age of the commercial air planes in Nigeria. Note as already observed above that the mean age of the commercial planes operating in Nigeria is estimated as 20.7 years. This observation may have informed the recent stipulation of a maximum age of 20 years as an eligibility criterion for any commercial plane to be allowed to operate in the country.

Thus the 20 years maximum age requirement for any commercial plane to be eligible to operate in Nigeria in effect implies that this upper age limit is approximately the mean age of the commercial planes in the country. This however does not enable the policy implementer have a bird's eye view and quickly and clearly determine the critical point and which planes must not be allowed to fly on the basis of age. Perhaps a more critical point to adopt and fix for this purpose would be the median age of the planes which is estimated as 19.4 years.

Hence strictly speaking if the median age is used, only the first 34 highest ranked commercial planes ranked in increasing order of age should be allowed to operate while the remaining lowest ranked 34 planes would not legitimately operate in the country.

Plane AA.8 with W_i of 35 ranked 17 aged 11.3 years and plane NA.5 with W_i of 33 ranked 18 aged 12.6 years and hence assigned a mean rank of 17.5 are estimated using the results of Table 2 and Eqn. 12 to be the planes that occupy the first quartile of the age distribution of the commercial planes. Thus these two planes with a mean rank of 17.5, the first quartile rank corresponding to 11.95 years are each younger than three fourths or about 50 ($f_i^- = 50$) and older than only one fourth, that is about 17($f_i^- = 17$) of all the commercial planes in the country.

The reverse is seen to be the case with planes AI.1 and AI.3 whose W_i values of -32 are each ranked 51.5. The age 22.3 years of these two planes is therefore the third quartile of the age distribution of the commercial airplanes in Nigeria and each is seen from Table 2 to be younger than only 17 planes ($f_i^+ = 17$), as old as only one other plane ($f_i^0 = 1$) and older than as many as 49 planes ($f_i^- = 49$).

Note that in all cases, positive values of W_i for any plane means that for that plane f_i^+ is greater than f_i^- so that the plane is relatively younger by the number of planes indicated by the value of W_i . The converse is the case when W_i is a negative value.

As already noted above, if the maximum age of 20 years recently mandatorily required by the air transport regulatory body for air planes to be able to operate in Nigeria is to be implemented, then it can be seen from the results of our analysis that this age may for real practical purposes approximated to correspond to the estimated median age of 19.4 years or 19 years for the commercial planes currently operating in the country. This will in effect mean that only about 33 or 34 of the commercial air planes currently using Nigeria's airspace are in fact eligible to operate in the country.

But even this allowable maximum age of 20 years approximated to a more easily operational age of 19 years may still be too high. Statistical tests for significance shows that the $W_i^{'s}$ indicated with asterisk (*) in Table 2 are statistically significant. Of particular interest are the 26 commercial planes with positive values of W_j that are statistically significant and the 26 commercial planes with negative values of W_i s that are also statistically significant. These later set of planes are each older than 21.2 years and hence younger than at most 23 other planes ($f_i^+ = 23$) and older than at least 43 other planes ($f_i^- = 43$) so that the relative gap in plane-years for each of these planes W_i is at most 20 and ranked 43.5 or lower, starting with planes AC9.6 and AD.2 should not be allowed to operate on the basis of age.

Strictly speaking also, based on the statistical significance of the difference between the planes by gaps in "plane years" W_i and for safety reasons if based on age only, the 26 commercial planes with positive W_i s that are statistically significant and aged at most 15.3 years and hence younger than at least 42 other planes ($f_i^+ = 42$) and older than at most 25 other planes ($f_i^- = 25$) so that the gap in "plane-years" W_i is at least 17 and ranked 26 or higher starting with plane NA.7 in the preferential ranking of the commercial planes should be qualified and eligible to operate in the country.

If need be, the commercial planes with gaps in plane years W_i that are not statistically significant may be allowed to operate even if temporarily starting with the plane with the highest value of W_i here of 15 aged at most 18.2 years (NA.9) down to the planes with the median age of 19.4 years, with $W_i = 0$ (CA.4 and CA.6). Thereafter those commercial planes with negative values of W_i that are not statistically significant may be included in the list of temporarily approved planes starting with those with the smallest negative values of W_i ($W_i = -4$, for CA.7 and CA.8) aged at most 19.9 years down to those planes with the highest non-significant negative values of W_i ($W_i = -15$ for CA.11 and AD.1) aged at most 20.9 years, if there is still need.

Finally it would be instructive to determine whether statistical difference exists between gaps in "plane years" W_i of planes with the smallest values of these indexes that are positive and statistically significant and those with the largest values of the indexes that are negative and significant; that is compare W_i of planes at the highest allowable age, that is planes with the smallest positive and statistically significant values of W_i and planes at the lowest allowable age, that is planes with the largest negative and statistically significant values of W_i.

To do this we note from Table 2 that the commercial plane with the smallest value of W_i that is positive and statistically significant is NA.7 aged 15.3 years with $W_i = 17$ ranked 26 and Var (W_i) = 62.687; while the planes with the largest value of W_i that is negative and statistically significant are AC9.6 and AD.2 each aged 21.2 years with $W_i = -20$ ranked 43.5 and Var (W_i) = 60.025.

Hence using Eqn. 11, we have that the corresponding test statistic is

$$\chi^2 = \frac{(W_1 - W_2)^2}{Var(W_1 - W_2)} = \frac{(17-(-20))^2}{62.687+60.025} = \frac{1369}{122.712} = 11.156$$

(P-value = 0.0008)

which is highly statistically significant. This result would seem to indicate that commercial planes that are 15.3 years old or less are likely to be more efficient and safe operationally in terms of age than planes that are 21.2 years or older and hence may be preferable as carriers.

These results seem to provide a strong indication that age is probably an important factor that should be considered in assessing the quality and air worthiness of planes.

4. SUMMARY AND CONCLUSION

We have employed a method developed for rank-ordering subjects on the basis of their relative performance in a trial or experiment for possible preferential selection when available resources may be limited to analyse data on the ages of commercial airplanes currently operating in Nigeria. The objective is to preferentially rank-order these planes by level of safety vis-à-vis their ages assuming age is an important factor affecting aircraft safety and air worthiness and also to assess the likely effectiveness of the recently announced twenty years maximum age eligibility criterion mandatory for all commercial planes using Nigeria's airspace.

It was found that the required maximum age of twenty years is only slightly less than the estimated mean age of 20.7 years of commercial planes currently operating in Nigeria. Altogether 37 commercial airplanes are 20 years or younger and would therefore normally be eligible to fly.

It was however found that a tighter cut-off critical point to use and one that is perhaps more statistically informed is the median age which is here estimated to be 19.4 years for the commercial planes. Under this criterion only 33 or 34 planes that are at most 19.4 years would be qualified while the remaining 34 older planes would not normally be allowed to continue flying.

Our analysis also shows that age is a statistically significant factor that should be considered in matters concerning reliability and quality of airplanes. Statistically significant differences gaps in relative "plane age" indexes are found to exist between planes with the smallest values of these indexes that are positive and statistically significant and those with the largest values of the indexes that are negative and also significant. In particular the difference between the plane age index at the highest allowable age (15.3 years) that is plane with the smallest positive and statistically significant value of "plane age" index and the "plane age" index of the plane at the lowest allowable age (21.2 years) that is plane with the largest negative and statistically significant value of "plane age" index is highly statistically significant. Hence because of the statistical significance of "plane years" here used as indexes of relative ages of commercial planes, only planes with positive and statistically significant plane age indices should be allowed to fly if operational safety of planes were to be based on age only. This would imply that the upper age limit for commercial plane flight eligibility in Nigeria would strictly speaking be set at 15.3 years or 15 years rather than 20 years as is presently the case. This would also in effect mean that only 26 commercial planes rather than 34 planes, if the age limit is set at the median age of 19.4 years; or 37 planes, if the required maximum age remains 20 years, would be qualified and eligible to fly Nigeria's air space.

But even if the required maximum age is relaxed to be up to 20 years then it is still found based on our analysis that only about 37 commercial air planes would be eligible to operate. It is also shown that the first 3 most highly rated planes are hence preferred and likely to be reliable and eligible in terms of age are AA.14, AA.13 and AA.12 in this order. The three lowest rated planes also in terms of "plane age" indexes are AK.2 and AK.5, Ak.4 and AC9.1; and AC9.2 and AC9.3 pairwise tied also in this order, and may not together with other commercial planes ranked 38 or lower normally be allowed to operate in the country if age is a determining factor of aircraft flight eligibility.

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