CSCanada
Advances in Petroleum Exploration and Development
Vol. 16, No. 1, 2018, pp. 44-48
DOI:10.3968/10681

ISSN 1925-542X [Print]
ISSN 1925-5438 [Online]
www.cscanada.net
www.cscanada.org

The Micro Vibration Equation of Rock and Its Analysis in Flat Indenter Basing on the Principle of Least Action

JI Guodong[a],[c]; WANG Haige[a]; LIU Li[a]; LI Siqi[b]*; SUN Liudang[d]; CHEN Zihe[b]

[a]CNPC Drilling Research Institute, Beijing, China.
[b]Petroleum Engineering College, Northeast Petroleum University, Daqing, Heilongjiang, China.
[c]Research Institute of Petroleum Exploration & Development, PetroChina, Beijing, China.
[d]CPTDC China Petroleum Technology & Development Corporation, Beijing, China.
*Corresponding author.

Supported by Scientific Research and Technology Development Project of CNPC (2016A-3902).

Received 12 July 2018; accepted 18 September 2018
Published online 26 September 2018

Abstract
Impact frequency of drill tools, vibration displacement of rock and other factors play a key role on the impact efficiency of vibration and rock breaking effect in the percussion drilling. In this paper, the micro vibration equation of rock in the impact of indenter was established based on the principle of least action. Then the relationship among vibration displacement of rock and quality and natural frequency of rock, impact force and impact frequency of indenter and time were analyzed. The results show that the curve of vibration displacement is kind of shape of cosine function, its size fluctuates up and down in the equilibrium position with the changes in various factors; The greater the impact of flat indenter is, The smaller the quality of rock is, the greater the vibration displacement of rock is; The closer the impact frequency of indenter and natural frequency of rock are, the greater the vibration amplitude of rock is, and it is significantly higher than the situation which the difference of impact frequency of indenter and natural frequency of rock is large.

Key words: Micro vibration equation; Natural frequency; Impact frequency; Vibration displacement; Principle of least action

Ji, G. D., Wang, H. G., Liu, L., Li, S. Q., Sun, L. D., & Chen, Z. H. (2018). The Micro Vibration Equation of Rock and Its Analysis in Flat Indenter Basing on the Principle of Least Action. *Advances in Petroleum Exploration and Development*, *16*(1), 44-48. Available from: http://www.cscanada.net/index.php/aped/article/view/10681
DOI: http://dx.doi.org/10.3968/10681

INTRODUCTION

Impacting has always been the main way for rock breaking during drilling. No matter rotary percussion drilling which is more mature or high frequency harmonic vibrating percussion drilling which is still in the stage of research or drilling by new drilling tools such as torsional impactor, are all closely related with the rock vibration by impacting (Jiang, Liu, Zhai, & Pan, 2006; Li, Yan, Li, & Zhang, 2013; Zhou, Zhang, & Zhang, 2012; Lv, Li, Wang, Wang, & Han, 2012).

Predecessors have done a lot of research on drill string vibration (Yan, Han, & Bi, 2006; Li, Zhang, Hou, Liu, & Xu, 2004; Liu, Ma, & Zhong, 2000), such as torsional vibration, vertical vibration, transverse vibration and so on. There are many scholars studying on the mechanical properties of rock under impacting (Li, Zuo, & Ma, 2005; Zhou et al., 2005; Liu, Xu, Li, & Li, 2010). There are also a lot of researchers simulating the process of drill tool impacting rock (Yin, Liu, Gao, & Sun, 2009; Kuang, Ma, Liu, & Wu, 2001). But in the present study, nobody establishes the analysis model of rock vibration under impacting. However, impact frequency of drill tools, vibration displacement of rock and other factors play a key role on the impact efficiency of vibration and rock breaking effect in the percussion drilling.

Therefore, according to the characteristics of rock vibration, use the principle of least action, establish the corresponding micro vibration model of rock. It can provide a theoretical basis for rock movement under impacting.

1. THE PRINCIPLE OF LEAST ACTION

The most general form of the movement of mechanical system can be described by the principle of least action. According to this principle, the characteristics of every mechanical system can be described by certain function,

$L(q_1, q_2, \cdots q_n, \dot{q}_1, \dot{q}_2 \cdots \dot{q}_n, t)$

It can be abbreviated as,

$L(q, \dot{q}, t)$

Assume that at the moment of $t=t_1$ and $t=t_2$, system has two determined location. It between the two places has the movement in the smallest possible value.

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t) dt \quad (1)$$

Where: q is generalized coordinates; $\dot{q}$ is generalized velocity; S is action; t is time and L is Lagrange function.

The principle of least action can also be written as,

$$\delta S = \delta \int_{t_2}^{t_1} L(q, \dot{q}, t) dt = 0 \quad (2)$$

2. THE MICRO VIBRATION MODEL OF ROCK

2.1 Vibration Equation in Multi-Degree of Freedom

The action of movement system is equation (1). Its variation form is,

$$\frac{d}{d_t} \frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} \quad (3)$$

According to the principle of least action in Lagrange, the motion equation of system is that makes equation (3) multi-time integral in the period of the movement and then takes a minimum value.

For vibration equation in multi-degree of freedom, the model is composed of a plurality of springs. Springs are not free vibration, there is interference between them, so the vibration exist coupling. We can describe the vibration equation as follows.

The general form of potential energy which is generated by deformation is,

$$U = \frac{1}{2} \sum k_{ik} x_i x_k \quad (4)$$

The general form of kinetic energy is,

$$T = \frac{1}{2} \sum m_{ik} \dot{x}_i \dot{x}_k \quad (5)$$

The coordinates of equilibrium position is defined as 0, so the minimum value of potential energy is 0. According to the symmetry of dynamics, we can get,

$$k_{ik}=k_{ki} \quad (6)$$

$$m_{ik}=m_{ki} \quad (7)$$

So the Lagrange's equation can be written as,

$$L(q, \dot{q}, t) = T - U \quad (8)$$

Basing on the additivity of Lagrange's equations, we can get,

$$L = T = \frac{1}{2} \sum m_{ik} \dot{x}_i \dot{x}_k - \frac{1}{2} \sum k_{ik} x_i x_k \quad (9)$$

From equation (9), we can get total differential equation of Lagrange,

$$dL = \frac{1}{2} \sum (m_{ik} \dot{x}_i \, d\dot{x}_k + m_{ik} \dot{x}_k \, d\dot{x}_i - k_{ik} x_i dx_k - k_{ik} x_k x_i) \quad (10)$$

The sum part in equation (10) is symmetric quadratic matrix, exchange i and j in equation above and consider the symmetry of m and k,

$$dL = \sum (m_{ik} \dot{x}_k \, d\dot{x}_i - k_{ik} x_k x_i) \quad (11)$$

According to the variational theory,

$$\sum m_{ik} \ddot{x}_k + \sum k_{ik} x_k = 0 \quad (12)$$

By the theory of fourier series, we can get,

$$x_k = A_k e^{iwt} \quad (13)$$

The determinant $\left| k_{ik} - \omega^2 m_{ik} \right| = 0$ can be obtained by taking equation (13) into equation (12). Obviously, ω is the eigenvalue for the corresponding matrix, namely the frequency of vibration.

2.2 The Micro Vibration Equation of Rock

Assume the rock is isotropic homogeneous medium, ignore the influence of formation pressure and temperature on the rock, the interaction of bit and rock is simplified to flat indenter and rock. Establish the micro vibration model of rock, the flat indenter is simulated by a plurality of springs stacked, as shown in figure 1. Divide the area under the indenter infinitely, extract a small infinitesimal to analyze separately, as shown in figure 2. Because of the force which each point of flat indenter does on the rock is the same in the acting surface, so the interaction relation between a point of flat indenter and the infinitesimal of rock can instead of it between flat indenter and rock. Consider the practical characteristics, ignore the coupling force and simplify the motion equation of multi-degree freedom vibration.

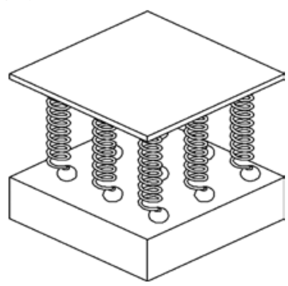

Figure 1
The Model of Rock Vibration

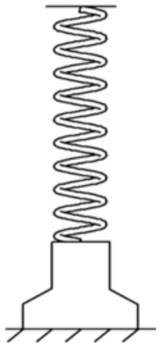

Figure 2
The Infinitesimal Analysis Unit of Flat Indenter and Rock

The applied energy from external force field can be regarded as a potential energy $U'(x,t)$. Make it power series expansion at x=0 and omit 2 orders infinitesimal,

$$U'(x,t) = U'(o,t) + x\frac{\partial U}{\partial x}\bigg|_{x=0} \tag{14}$$

The first item is the total derivative of time, it will disappear at the time of variation. We can get Lagrange's equation based on the definition of potential energy.

$$L = \frac{m\dot{x}^2}{2} - \frac{kx^2}{2} + xF(t) \tag{15}$$

The corresponding differential equation after variation is,

$$m\ddot{x} + kx = F(t) \tag{16}$$

The force which is applied by indenter is periodic, so it can be described as follows,

$$F = f\cos(w_1 t + \beta) \tag{17}$$

Where: ω_1 is the vibration frequency of indenter.

Take equation (17) into (16), its general solution is got,

$$x = a\cos(wt + \alpha) + \frac{f}{m(w^2 - w_1^{\,2})}\cos(w_1 t + \beta) \tag{18}$$

Where: x is the vibration displacement of rock, ω is the natural frequency of rock, m is the mass of rock, α,β are initial phase, a is the amplitude.

What we analyze is infinitesimal, so m actually represents surface density multiply by ds. Because the vibration state of the whole rock is the same under the flat indenter, so the vibration state of infinitesimal can express the whole region. Equation (18) is the vibration equation of rock under the flat indenter.

3. FACTORS ANALYSIS OF MICRO VIBRATION MODEL OF ROCK

Basing on the micro vibration model of rock, we know that the displacement caused by flat indenter impacting rock is related to the natural frequency of rock, the impact frequency of indenter, the mass of rock, the impact force and time. Analyze the influence law of various factors on the rock vibration by drawing relationship curves among various factors and vibration displacement.

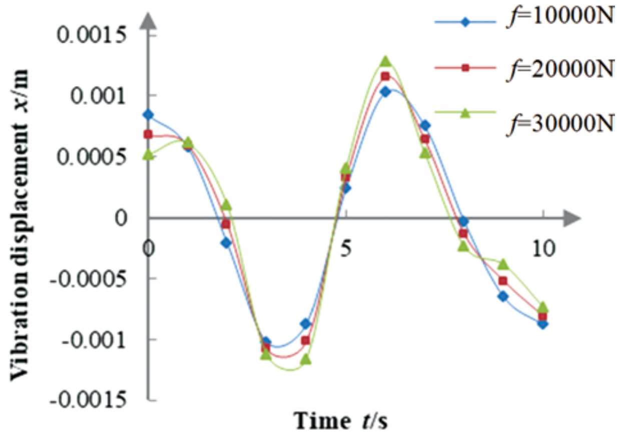

Figure 3
The Relationship Between Vibration Displacement and Time in the Different Impact

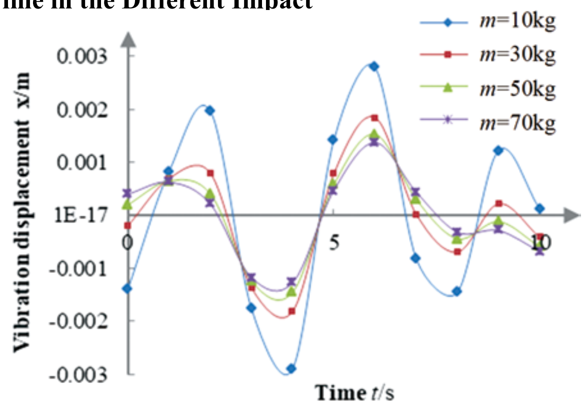

Figure 4
The Relationship Between Vibration Displacement and Time in the Different Quality of Rock

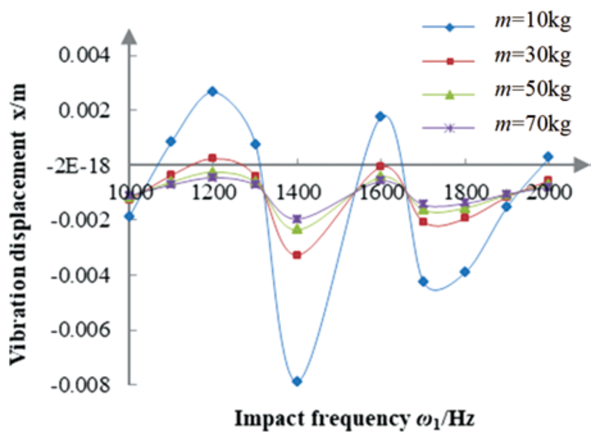

Figure 5
The Relationship Between Vibration Displacement and Impact Frequency in Different Quality of Rock

The path of rock vibration displacement is the up and down reciprocating curve which is composed of two cosine functions. The initial phases of two cosine functions are set to 0 to consider the rock vibration. Figure 3 shows that the greater the impact force of flat indenter is, the greater the maximum displacement of rock is. Figure 4 and Figure 5 show that the smaller the mass of rock is, the greater the vibration displacement is. It is more prone to vibration and to be broken. In the actual situation, the vibration amplitude of rock is very weak, so the variation range of maximal displacement is also very small in Figures.

Impact rock with flat indenter in a frequency of 1000 Hz and the vibration displacement curve is shown in Figure 6. It shows that the rock whose natural frequency is 1300 Hz can be caused the maximum vibration displacement. Impact the rock whose natural frequency is 2000 Hz, its vibration displacement curve is shown in Figure 7. It shows that the rock can be caused maximum displacement when the impact frequencies are 1500Hz and 2500Hz. Figure 8 shows that the rocks whose natural frequency is 1000 Hz, 1500 Hz and 2000 Hz can be caused maximum displacement at the impact frequency are 1000Hz, 1500Hz and 2000Hz respectively. What's more, the maximum displacements are significantly higher than ones in the other impact frequencies.

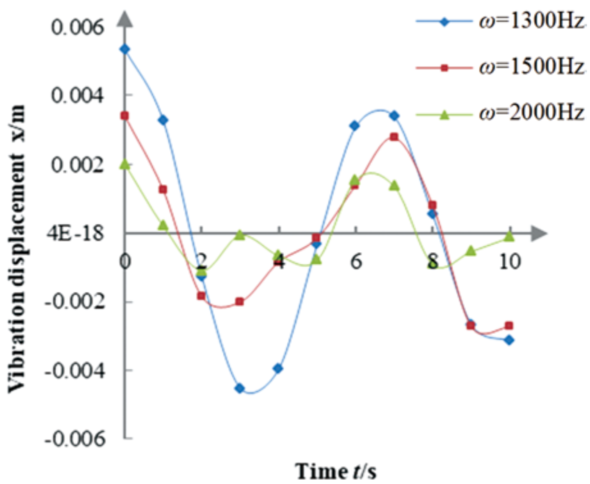

Figure 6
The Relationship Between Vibration Displacement and Time in Different Natural Frequency of Rock

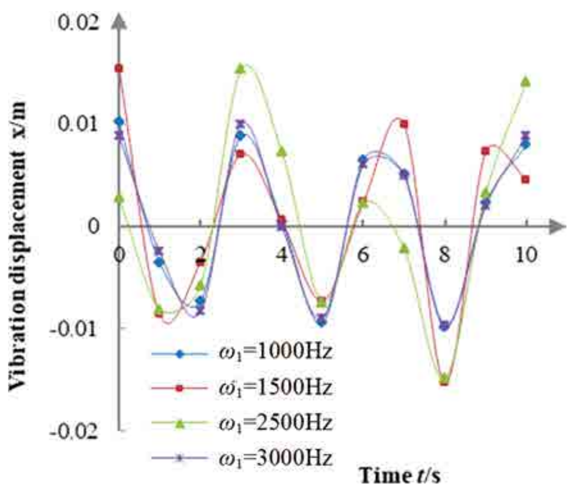

Figure 7
The Relationship Between Vibration Displacement and Time in Different Impact Frequency

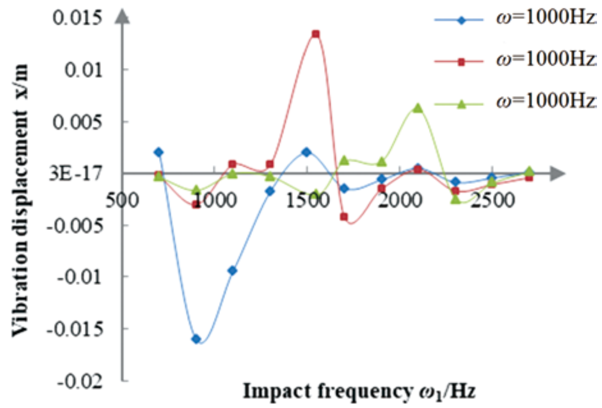

Figure 8
The Relationship Between Vibration Displacement and Impact Frequency in Different Natural Frequency of Rock

Therefore, the closer the natural frequency of rock and the impact frequency of indenter are, the greater the vibration displacements of rock are and vice versa. When the natural frequency of rock is far away from the impact frequency of indenter, the curves appear unstable fluctuation, it means that the vibration of rock is in a state of instability. So we should choose the impact frequency which is close to the natural frequency as far as possible when the rock is broken. At this time, rock can be achieved resonance state, the vibration amplitude is the largest and easy to be broken.

Based on the above analysis of factors on the micro vibration model of rock, we know that the mass of rock and the impact force of flat indenter have less impact on vibration displacement, while the natural frequency of rock and the impact frequency of indenter have obvious effects on vibration characteristics of rock. In addition, due to the micro vibration displacement curve of rock is kind of cosine function, so the size of vibration displacement changes with factors periodically which corresponds to the characteristic of rock vibrating at the equilibrium position.

CONCLUSIONS

Based on the principle of least action, the micro vibration model of rock is established and the displacement equation is given. The micro vibration displacement curve of rock is kind of cosine function, the size of vibration displacement changes with factors ups and downs at $x=0$ which corresponds to the characteristic of rock vibrating at the equilibrium position in the actual situation.

When the natural frequency of rock and the impact frequency of flat indenter are certain, the smaller the mass of rock is, the easier the rock to be vibrated, the greater the impact force is, the larger the vibration displacement is.

When the mass of rock and the impact force of flat indenter are certain, the closer the natural frequency of rock and the impact frequency of indenter are, the greater the vibration displacements of rock are, and significantly higher than the situation which the difference between them is big.

REFERENCES

Jiang, H. W., Liu, Y. S., Zhai, Y. H., & Pan, J. (2006). Research on rock-crushing mechanics model in rotary and percussive drilling. *Oil Drilling Technology, 31*(1), 10-16.

Kuang, Y. C., Ma, D. K., Liu, Q. Y., & Wu, Z. B. (2001). Computer simulation on dynamic action of drillstring-bit-rock system. *Acta petrolei sinica, 22*(3), 81-85.

Li, W., Yan, T., Li, S. Q., & Zhang, X. N. (2013). Rock fragmentation mechanisms and an experimental study of drilling tools during high-frequency harmonic vibration. *Petroleum Science*, (10), 205-211.

Li, X. B., Zuo, Y. J., & Ma, C. D. (2005). Failure criterion of strain energy density and catastrophe theory analysis of rock subjected to static-dynamic coupling loading. *Rock and Soil Mechanics, 24*(16), 2814-2823.

Li, Z. F., Zhang, Y. G., Hou, X. T., Liu, W. D., & Xu, G. Q. (2004). Analysis of longitudinal and torsion vibration of drillstring. *Engineering Mechanics, 21*(6), 203-209.

Liu, J. F., Xu, J., Li, Q. S., & Li, G. L. (2010). Experimental research on damping parameters of rock under cyclic loading. *Chinese Journal of Rock Mechanics and Engineering, 29*(5), 1036-1041.

Liu, Q. Y., Ma, D. K., & Zhong, Q. (2000). A drilling string torsional vibration model and solution. *Acta Petrolei Sinica, 21*(2), 78-82.

Lv, X. P., Li, G. X., Wang, Z. Y., Wang, S., & Han, Y. P. (2012). Experiment of torkbuster on Well YS1 in silurian formation. *Oil Drilling & Production Technology, 3*(42), 99-101.

Yan, T., Han, C. J., & Bi, X. L. (2006). The finite element analysison axial vibration of drill string in deviated wells. *Petroleum Drilling Techniques, 34*(4), 5-8.

Yin, P., Liu, Y. C., Gao, K., & Sun, Z. G. (2009). Study on dynamic simulation research of single drill pipe based on finite element method. *Oil Drilling & Production Technology, 31*(4), 12-20.

Zhou, X. L., Zhang, J. C., & Zhang, D. Q. (2012). Experimental application of TorkBuster torsional impactor in Yuanba region. *Drilling & Production Technology, 35*(2), 15-17.

Zhou, X. P., Ha, Q. L., … Zhang, Y. X. (2005). Analysis of localization of deformation and complete stress-strain relation for mesoscopic heterogenous brittle rock material when axial stress is held constant while lateral confinement is reduced. *Rock and Soil Mechanics, 24*(18), 3236-3245.

