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Productivity Prediction for Stacked Multilateral Horizontal Well Under Open Hole Series Completion Methods

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Abstract

Productivities of stacked multilateral horizontal well under open hole series completion methods were predicted by an analytical model. The analytical model was established using conformal transformation, mirror image, potential superposition and equivalent flow resistance. The ideal well's formula will be simplified to the famous Borisove's formula when stacked well has only one branch and locate it in middle vertical depth of reservoir. Several productivity influencing factors were analyzed to provide references for stacked well completion design. Case studies show that, stacked well productivity decreases with higher screen filtration precision; Conventional horizontal well productivity is more sensitive to filtration precision than that of stacked well; Stacked well's vertical location in reservoir with upper and lower sealed boundary has little impact on productivity; Analytical model overestimates productivity because of ignorance of seepage disturbance and well bore flow pressure drop, an infinitesimal sectional model works as a correction model and a correction coefficient is obtained to effectively reduce the error of analytical model.

Key words: Stacked well; Open hole; Analytical model; Mirror image; Potential superposition; Seepage resistance

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INTRODUCTION

Multilateral horizontal well has larger reservoir drainage area than conventional horizontal well, it is used to further enhance economic performance of oil and gas field development, and has become an important petroleum technology direction[1-2]. Notably, predicting multilateral well productivity accurately will ensure a high efficiency development, and has received attention in petroleum engineering around the world[3-13]. Various types of multilateral horizontal wells, such as radial type and herringbone type, have their productivities predicted, however, the stacked type[14] was left, even many multilateral wells in field production can be treated as this type to calculate productivities. Then a specific model using complex variable function theory and fluid mechanics in porous medium is presented to predict productivity of stacked well under open hole series completion methods, its influencing factors were analyzed subsequently. The research in this paper provides rapid productivity prediction method for stacked well completion design.

1. IDEAL WELL PRODUCTIVITY

1.1 Physical Model

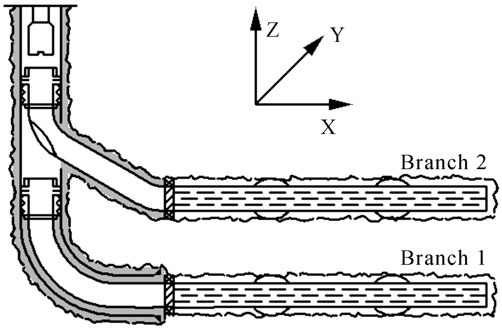

Figure 1
Stacked Multilateral Horizontal Well

Figure 1 shows schematic diagram of stacked multilateral horizontal well as well as its coordinate system. The stacked well located in a infinite reservoir which is supplied by constant potential boundary around but covered by impermeable boundary up and down. Formation and fluid properties are assumed to be independent of pressure, and only single phase oil is considered in analytical model. Horizontal production segment of each branch is set to be equal. The seepage issue solved in three dimensional space XYZ is decomposed into two seepage issues solved in two dimensional planes XY and YZ, according to pseudo three dimensional solving method. As shown in Figures 2 and 3, external seepage resistance is handled in Plane XY while the internal one is handled in Plane YZ.

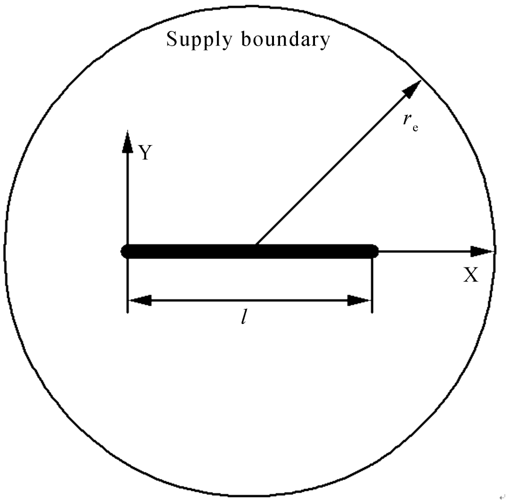

Figure 2
Plant XY

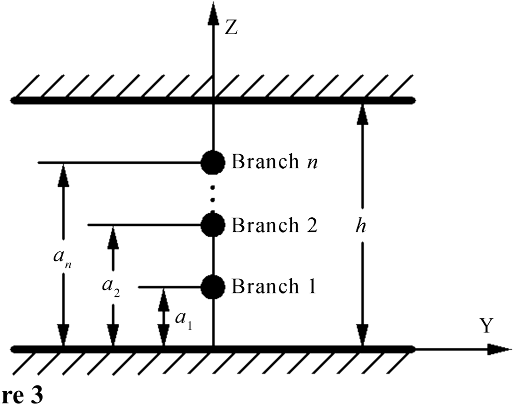

Figure 3
Plane YZ

1.2 Mathematical Derivation of External Seepage Resistance

During external seepage resistance solution in Plane XY, stacked multilateral horizontal well should be simplified as an imaginary fracture whose height is equal to reservoir thickness and length equal to stacked well's production interval length. Conformal transformation is used to map Plane XY to a new complex plane where the analytical method can solve the issue easily, as shown in Figure 4. Production invariance principle during conformal transformation ensures external resistance unchanged before and after mapping, that is the external resistance in Plane XY will be obtained when that of new complex plane is solved, which is known as the equivalent seepage resistance law. Use the conformal transformation formula:

$$z-\frac{l}{2}=\frac{l}{2}\mathrm{ch}w \qquad (1)$$

Where z = a point in Complex Plane XY, $z = x + iy$; w = a point in Complex Plane UV, $w = u + iv$; l = lateral length of stacked well, m.

All of the points in Complex Plane XY are mapped to a strip region in Complex Plane UV, and the original imaginary fracture in Plane XY is mapped to a new one in Plane UV, which is shown in Figure 4. Note that Boundary $v = \pi$ and Boundary $v = 0$ are sealed, and the new fracture rotates 90° from the original one.

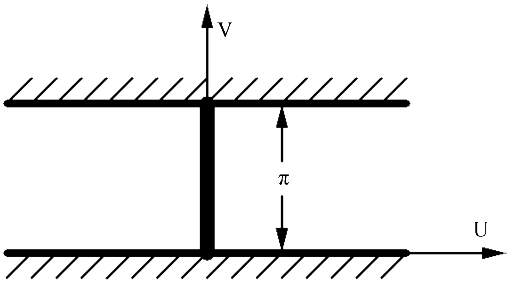

Figure 4
Complex Plane UV

Any point located in supply boundary r_e in Plane XY has the following relationship with that in Plane UV, when r_e is large enough:

$$u_e=\ln\left(\frac{4r_e}{l}\right) \qquad (2)$$

Where u_e = supply boundary in Plane UV corresponding to r_e in Plane XY.

Taking a strip reservoir at any side of imaginary fracture in Plane UV, and treating the fluid flow as Darcy flow, the relationship of production vs. pressure difference can be described by

$$Q_S=K_h\frac{A\Delta p}{u_e\mu_O} \qquad (3)$$

Where Q_S = strip reservoir oil production at any side of imaginary fracture in Plane UV, m^3/ks; K_h = horizontal permeability, μm^2; A = cross sectional area of strip reservoir in Plane UV, m^2; Δp = production pressure difference, MPa; μ_O = oil viscosity, mPa·s.

Strip reservoirs at both sides of imaginary fracture in Plane UV are symmetrical to each other, so we have:

$$Q_T=2Q_S \qquad (4)$$

Where Q_T = total oil production of stacked well, m^3/ks.

Combining Formulas (2), (3) and (4), external seepage resistance can be described by:

$$R_{\mathrm{EX}} = \frac{\mu_{\mathrm{O}}}{2\pi K_{\mathrm{h}} h}\ln\frac{4r_{\mathrm{e}}}{l} \tag{5}$$

Where h = reservoir thickness, m.

1.3 Mathematical Derivation of Internal Seepage Resistance

As shown in Figure 3, horizontal production section of each branch should be treated as point sink in internal seepage resistance solution in Plane YZ. Using mirror image and potential superposition, and according to mathematical treatment reported in Reference [2], internal seepage resistance of branch i can be written as:

$$R_{\mathrm{IN}i} = \frac{\mu_{\mathrm{O}}}{2\pi K_{\mathrm{h}} l}\ln\frac{h}{2n r_{\mathrm{W}}\sin\frac{\pi a_i}{h}} \tag{6}$$

Where n = branch number; r_{W} = well bore radius, m; a_i = the distance between branch i and reservoir lower sealed boundary, m, i = 1,2,3,...

Total internal seepage resistance is a parallel connection of those of each branch, ignoring seepage flow disturbance among branches. Considering influences of formation anisotropy and crude oil volume shrinkage, total internal seepage resistance R_{IN} can be written as:

$$R_{\mathrm{IN}} = \frac{\mu_{\mathrm{O}} B_{\mathrm{O}}}{2\pi K l}\left(\sum_{i=1}^{n}\ln^{-1}\frac{\beta h}{2\pi r_{\mathrm{W}}\sin\frac{\pi a_i}{h}}\right)^{-1} \tag{7a}$$

$$K = \sqrt{K_{\mathrm{h}} K_{\mathrm{v}}} \tag{7b}$$

$$\beta = \sqrt{\frac{K_{\mathrm{h}}}{K_{\mathrm{v}}}} \tag{7c}$$

Where B_{O} = oil volume factors, dimensionless; β = reservoir anisotropy coefficient, dimensionless; K = average permeability, μm^2; K_{v} = vertical permeability, μm^2.

1.4 Productivity Formulation

Ideal well production doesn't have any additional seepage resistance, therefore, the total seepage resistance is the summation of external resistance and internal resistance, ideal well productivity can be written as:

$$J_{\mathrm{IDE}} = \frac{2\pi K_{\mathrm{h}} h/\left(\mu_{\mathrm{O}} B_{\mathrm{O}}\right)}{\ln\frac{4r_{\mathrm{e}}}{l} + \frac{\beta h}{l}\left(\sum_{i=1}^{n}\ln^{-1}\frac{\beta h}{2\pi r_{\mathrm{W}}\sin\frac{\pi a_i}{h}}\right)^{-1}} \tag{8}$$

When there exists only one lateral and locate it in the middle vertical depth of reservoir, the stacked well is simplified to a conventional horizontal well, that is n = 1 and a_1 = h/2, according to Formula (8), its productivity can be written as:

$$J_{\mathrm{IDE}} = \frac{2\pi K_{\mathrm{h}} h/\left(\mu_{\mathrm{O}} B_{\mathrm{O}}\right)}{\ln\frac{4r_{\mathrm{e}}}{l} + \frac{\beta h}{l}\frac{\beta h}{2\pi r_{\mathrm{W}}}} \tag{9}$$

Formula (9) is the famous Borisov's Formula which becomes just a special form of Formula (8) derived in this paper.

2 PRODUCTIVITY UNDER OPEN HOLE SERIES COMPLETION METHODS

2.1 Actual Open Hole Well Productivity

Actual wells always suffer from drilling and completion damage, which bring additional seepage resistance, then the total seepage resistance can be written as:

$$R_{\mathrm{T}} = R_{\mathrm{EX}} + R_{\mathrm{IN}} + R_{\mathrm{D}} \tag{10}$$

R_{D} should be treated as a part of internal seepage resistance, because formation damage only exists around well bore. Then the new internal seepage resistance R_{IND} is the combination of R_{IN} and R_{D}. For branch i, R_{IND} can be written as:

$$R_{\mathrm{IND}i} = \frac{\mu_{\mathrm{O}} B_{\mathrm{O}}}{2\pi K_{\mathrm{h}} h}\left(\frac{\beta h}{l}\ln\frac{\beta h}{2\pi r_{\mathrm{W}}\sin\frac{\pi a_i}{l}} + S_{\mathrm{hD}}\right) \tag{11}$$

Where S_{hD} = skin factor of formation damage, dimensionless, which can be calculated according to Reference [15].

Total internal seepage resistance can be written as:

$$R_{\mathrm{IND}} = \frac{\mu_{\mathrm{O}} B_{\mathrm{O}}}{2\pi K_{\mathrm{h}} h}\left[\sum_{i=1}^{n}\left(\frac{\beta h}{L}\ln\frac{\beta h}{2\pi r_{\mathrm{W}}\sin\frac{\pi a_i}{h}} + S_{\mathrm{hD}}\right)^{-1}\right]^{-1} \tag{12}$$

Then actual well productivity can be described by:

$$J_{\mathrm{ACT}} = \frac{2\pi K_{\mathrm{h}} h/\left(\mu_{\mathrm{O}} B_{\mathrm{O}}\right)}{\ln\frac{4r_{\mathrm{e}}}{l} + \left[\sum_{i=1}^{n}\left(\frac{\beta h}{l}\ln\frac{\beta h}{2\pi r_{\mathrm{W}}\sin\frac{\pi a_i}{h}} + S_{\mathrm{hD}}\right)^{-1}\right]^{-1}} \tag{13}$$

2.2 Productivity of Actual Wells Under Sand Control Completion Methods

When stacked well bore is completed with open hole sand control screen pipe, for example, wire wrapped linear, reservoir sand will accumulate in annular space between screen pipe and well bore wall in production, and then form an accumulation layer which will offer another additional seepage resistance R_{S} and reduce oil productivity. Then total seepage resistance R_{T} can be described by:

$$R_T = R_{EX}+R_{IN}+R_D+R_S \tag{14}$$

Because R_S is limited to annular space, it also should be treated as a part of internal seepage resistance, the new internal seepage resistance is the combination of R_{IN}, R_D and R_S. Following the processing method in above section, the productivity of actual stacked well under sand control completion method can be written as:

$$J_S = \frac{2\pi K_h h / \left(\mu_O B_O\right)}{\ln\frac{4r_e}{l} + \left\{ \sum_{i=1}^{n} \left[\frac{\beta h}{l} \left(\ln \frac{\beta h}{2\pi r_W \sin\frac{\pi a_i}{h}} + S_S \right) + S_{hD} \right]^{-1} \right\}^{-1}} \tag{15}$$

Where S_S = skin factor caused by sand control completion, dimensionless, which can be calculated according to Reference [15].

2.3 Influence of Completion Method on Productivity

In this section, results of several examples under different completion methods are discussed. Reservoir and fluid parameters are listed in Table 1, which work as basic parameters in all the examples in this paper.

Table 1
Basic Reservoir and Fluid Parameters

Parameter	Value
Branch number	3
Lateral production interval length, m	200.0
Well bore diameter, mm	215.8
Reservoir thickness, m	20.0
Vertical distance between branch 1 And reservoir bottom boundary, m	5.0
Vertical distance between branch 2 And reservoir bottom boundary, m	10.0
Vertical distance between branch 3 And reservoir bottom boundary, m	15.0
Supply boundary radius, m	500.0
Horizontal permeability, μm^2	0.3
Vertical permeability, μm^2	0.1
Oil viscosity, mPa·s	9.0
Oil density, g/cm^3	0.85
Oil volume coefficient	1.15
Screen pipe outer diameter, mm	127.0
Production pressure difference, MPa	1.0

Each sand control completion method has specific filter fineness, which correspond to different S_S and then cause different productivity. The calculated productivities under different open hole series completion methods are shown in Table 2.

Table 2
Productivities Under Open Hole Series Completion Methods

Completion method	Ideal open hole	Actual open hole	Wire wrapped screen	Sand control screen wrapped with precise micro porous fabric	Sand control screen wrapped with metal fiber filling layer
Stacked well productivity (3 branches), $m^3/(d\cdot MPa)$	10.25	9.43	6.87	5.54	4.41
Productivity ratio (compared with ideal stacked well)	1.00	0.92	0.67	0.54	0.43
Horizontal well productivity	6.53	5.58	3.22	2.35	1.67
Productivity ratio (compared with ideal horizontal well)	1.00	0.85	0.49	0.36	0.26

Table 1 shows that, completion method has obvious influence on productivity, higher screen filtration precision reduces the permeability of formation sand accumulation layer in annular space between screen and well bore wall, when additional seepage resistance becomes higher, the productivity losses more. Table 1 also discusses conventional horizontal well under different completion methods. Productivity ratios compared with ideal well indicate that stacked well productivity is less sensitive to filtration precision than that of horizontal well, therefore, stacked well shows its advantage in open hole series completions with high filtration precision screen.

3. ANALYSES ON PRODUCTIVITY INFLUENCING FACTORS

Taking wire wrapped screen completion method for example, the influences of stacked well vertical position, horizontal production interval length, branch number and production pressure difference on stacked well productivity are discussed in this section. The results calculated by analytical model are compared with those by infinitesimal section model. Table 3 shows how productivity changes with stacked well vertical location in reservoir.

Table 3
Influence of Vertical Location on Productivity

Distance between branch i and sealed reservoir bottom boundary, m			Productivity, $m^3/(d\cdot MPa)$	
Branch 1	**Branch 2**	**Branch 3**	**Analytical model**	**Infinitesimal section model**
1.0	6.0	11.0	6.31	4.79
3.0	8.0	13.0	6.85	5.53
5.0	10.0	15.0	6.87	5.84
7.0	12.0	17.0	6.85	5.53
9.0	14.0	19.0	6.31	4.79

Table 3 shows that, example wells' vertical locations in reservoir have little impact on productivity, a number of calculation examples indicate that this rule applies to all the stacked wells unless branches are drilled very close to each other.

Figures 5, 6 and 7 show how productivity changes with horizontal production interval length, branch number and production pressure difference, respectively.

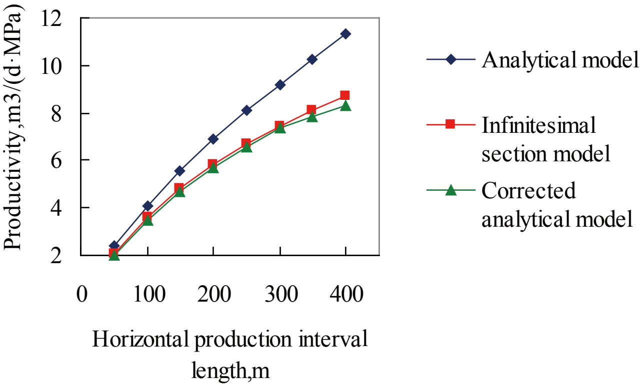

Figure 5
Influence of Horizontal Production Interval Length on Productivity

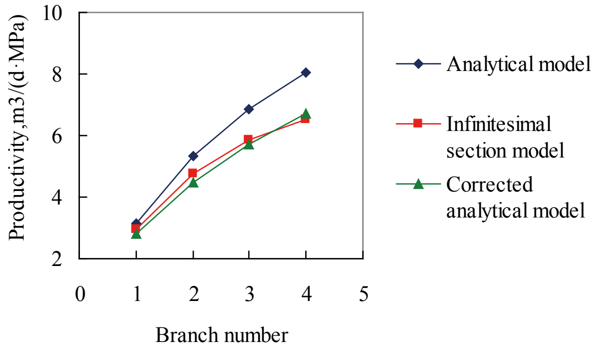

Figure 6
Influence of Branch Number on Productivity

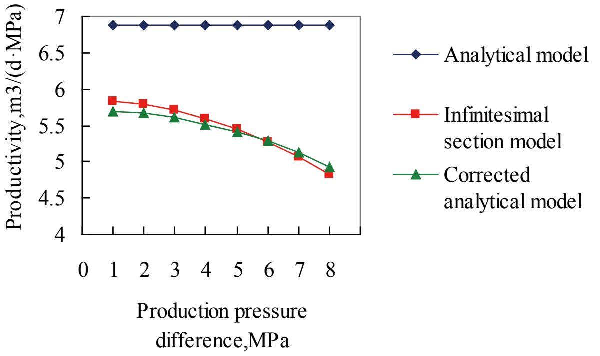

Figure 7
Influence of Production Pressure Difference on Productivity

Example results calculated by two models show that productivity increases with horizontal production interval length and branches number, but increment speed slows down. The analytical model productivity remain unchanged vs. production pressure difference while that by infinitesimal section model decreases with growing pressure difference.

Results calculated by analytical model are higher than those by infinitesimal section model, because analytical model ignores seepage disturbance and well bore flow pressure drop, both of which will reduce productivity, so its results are overestimated. Analytical error becomes larger with longer production interval, more branches and larger production pressure difference, then infinitesimal section model works as a correction model. A correction coefficient is obtained by fitting a large number of calculation data in model runs to reduce the error, which can be used by multiplying analytical productivity and written as:

$$C = 0.893e^{-31.256\gamma} + 0.107 \tag{16a}$$

$$\gamma = \frac{nQ_T l^2}{n_C Q_{MAX} r_e h} + \sum_{i=2}^{n} \left| \frac{a_{i-1} - a_i}{h} - \frac{1}{n+1} \right| \tag{16b}$$

Where n_C = reference value of branch number, n_C=8;

Q_{MAX} = absolute open flow of the whole stacked well, m^3/ks.

The corrected analytical results, which fit infinitesimal section results quite well, are shown in Figures 5, 6 and 7.

CONCLUSIONS

Productivity formulas of stacked multilateral horizontal well under open hole series completion methods were derived using pseudo three dimensional solving method, and the following conclusions have been drawn:

(a) When stacked well has only one branch and locate it in reservoir's middle vertical depth, ideal stacked well productivity formula will be simplified to the famous Borisov's formula, which becomes a special form of ideal stacked well productivity;

(b) Completion method has obvious influence on stacked well productivity which decreases as screen filtration precision becomes higher, moreover, conventional horizontal well productivity is more sensitive to filtration precision than that of stacked well. Stacked wells vertical location in reservoir with upper and lower sealed boundary has little impact on productivity unless branches are drilled very close to each other;

(c) Productivity increases with horizontal production interval length and branches number, but increment speed slows down, which indicates that overlong horizontal production interval and too many branches do not show any advantages in oil field development because of economic benefits;

(d) Analytical model ignores seepage disturbance and well bore flow pressure drop, so its results is overestimated. The error, taking infinitesimal section model for correction, becomes larger with longer horizontal production interval, more branches and larger production pressure difference, but the error can be reduced effectively by the correction coefficient obtained in this paper.

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