

A New Specific Property of Schweizer-Sklard Operators

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Abstract: A new specific property of Schweizer-Sklard operators is discovered that Schweizer-Sklard operations are continued fuzzy operators. The relation between Schweizer-Sklard operators and other Fuzzy operators is analysis. The conclusion that Schweizer-Sklard operations could partly replace other fuzzy operations most in use is given. At the same time the relation between Schweizer-Sklard operations and Zadeh operations as well as probability operators is given.

Keywords: Operator; Fuzzy operator; Schweizer-Sklard operator; Continued fuzzy operator

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INTRODUCTION

Since Zadeh established Fuzzy set theorem in 1965, he introduced a pair of Fuzzy operators. They are named Zadeh operators^[1]. There is a lot of discussion about the fuzzy operators^[2,3,4,5,6,7,8,9,10], but there is little discussion about the Schweizer-Sklard operators. This paper discussed membership relation in Schweizer-Sklard, Zadeh operator and other generalized operator in common use.

1. PRIMARY DEFINITIONS OF SCHWEIZER-SKLARD OPERATORS

We introduce a pair of important operators, Schweizer-Sklard operators, whose primary definition are as follows^[4].

Definition 1.1 $\forall \tilde{A}, \tilde{B}, \tilde{C} \in P(U), \forall u \in U, p \in (\mathbb{R}, +\infty)$. Schweizer-Sklard product $\tilde{C}(u)$ of $\tilde{A}(u)$ and $\tilde{B}(u)$, denoted $\tilde{C}(u) = \tilde{A}(u) \dot{s}s \tilde{B}(u)$, is defined by

$$\tilde{A}(u) \dot{s}s \tilde{B}(u) = \begin{cases} 0 & \text{if } p > 0, \tilde{A}(u) = 0 \text{ or } \tilde{B}(u) = 0 \\ [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} & \text{if } p > 0, \tilde{A}(u) \bullet \tilde{B}(u) \neq 0 \\ \tilde{A}(u) \bullet \tilde{B}(u) & \text{if } p = 0 \\ [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} & \text{if } p < 0, \tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} > 1 \\ 0 & \text{if } p < 0, \tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} \leq 1 \end{cases} \quad (1.1)$$

Definition 1.2 $\forall \tilde{A}, \tilde{B}, \tilde{C} \in P(U), \forall u \in U, p \in (-\infty, +\infty)$. Schweizer-Sklard sum $\tilde{C}(u)$ of $\tilde{A}(u)$ and $\tilde{B}(u)$, denoted $\tilde{C}(u) = \tilde{A}(u) \overset{+}{ss} \tilde{B}(u)$, is defined by

$$\tilde{A}(u) \overset{+}{ss} \tilde{B}(u) = \begin{cases} 1 & \text{if } p > 0, \tilde{A}(u) = 1 \text{ or } \tilde{B}(u) = 1 \\ 1 - \{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}^{-1/p} & \text{if } p > 0, \tilde{A}(u) \bullet \tilde{B}(u) \neq 1 \\ \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u) \bullet \tilde{B}(u) & \text{if } p = 0 \\ 1 - \{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}^{-1/p} & \text{if } p < 0, [1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} > 1 \\ 0 & \text{if } p < 0, [1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} \leq 1 \end{cases} \quad (1.2)$$

The pair of operators $(\overset{+}{ss}, \overset{+}{ss})$ is called Schweizer-Sklard operators.

2. A NEW IMPORTANT SPECIFIC PROPERTY OF SCHWEIZER-SKLARD OPERATORS

On property of Schweizer-Sklard operators, the paper^[4] has proved that formula (1.1) and (1.2) are monotone functions for variables $\tilde{A}(u)$ or $\tilde{B}(u)$.

But a new important specific property of Schweizer-Sklard operators has not mentioned up to now.

We give a new conclusion as follows:

Theorem 2.1 When $p > 0$, formula (1.1) is monotone increasing function for the parameter p .

Proof. $\forall u \in U$, if $p > 0, \tilde{A}(u) \bullet \tilde{B}(u) \neq 0$, let

$$F(p) = [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} = \exp\left\{\frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p}\right\}$$

then

$$\begin{aligned} \frac{\partial}{\partial p} F(p) &= F(p) \bullet \frac{\partial}{\partial p} \left\{ \frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p} \right\} = F(p) \bullet G(p) \\ G(p) &= \frac{[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1] \ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1] - \tilde{A}(u)^{-p} \ln \tilde{A}(u)^{-p} - \tilde{B}(u)^{-p} \ln \tilde{B}(u)^{-p}}{p^2 [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]} \end{aligned}$$

Let $x = \tilde{A}(u)^{-p}, y = \tilde{B}(u)^{-p}$ then

$$G(p) = \frac{(x + y - 1) \ln(x + y - 1) - x \ln x - y \ln y}{p^2(x + y - 1)} = \frac{1}{p^2} \ln[(x + y - 1)x^{1-\frac{1}{x}+\frac{y}{x}}y^{1-\frac{1}{y}+\frac{x}{y}}].$$

Because $\tilde{A}(u) \in (0, 1]$ and $\tilde{B}(u) \in (0, 1]$, then $x = \tilde{A}(u)^{-p} \in [1, +\infty)$ and $y = \tilde{B}(u)^{-p} \in [1, +\infty)$. this means $x + y - 1 \geq 1, y/x > 0, x/y > 0, 1-1/x \geq 0, 1-1/y \geq 0$. We have

$$1-1/x + y/x > 0, 1-1/y + x/y > 0$$

then

$$x^{1-\frac{1}{x}+\frac{y}{x}} \geq 1, y^{1-\frac{1}{y}+\frac{x}{y}} \geq 1$$

This means $G(p) \geq 0$. Because $F(p) > 0$, we have

$$\frac{\partial}{\partial p} F(p) = F(p) \bullet \frac{\partial}{\partial p} \left\{ \frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p} \right\} = F(p) \bullet G(p) \geq 0$$

This means that when $p > 0$, formula (1.1) is monotone increasing function for the parameter p .

Theorem 2.2 When $p < 0$, formula (1.1) is monotone decreasing function for the parameter p .

Proof. $\forall u \in U$, if $p < 0$, $\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} > 1$, let

$$F(p) = [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} = \exp\left\{\frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p}\right\}$$

then

$$\begin{aligned} \frac{\partial}{\partial p} F(p) &= F(p) \bullet \frac{\partial}{\partial p} \left\{ \frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p} \right\} = F(p) \bullet G(p) \\ G(p) &= \frac{[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1] \ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1] - \tilde{A}(u)^{-p} \ln \tilde{A}(u)^{-p} - \tilde{B}(u)^{-p} \ln \tilde{B}(u)^{-p}}{p^2[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]} \end{aligned}$$

Let $x = \tilde{A}(u)^{-p}$, $y = \tilde{B}(u)^{-p}$ then

$$G(p) = \frac{(x + y - 1) \ln(x + y - 1) - x \ln x - y \ln y}{p^2(x + y - 1)} = \frac{1}{p^2} \ln[(x + y - 1)x^{1-\frac{1}{x}+\frac{y}{x}}y^{1-\frac{1}{y}+\frac{x}{y}}]$$

Because $\tilde{A}(u) \in (0, 1]$ and $\tilde{B}(u) \in (0, 1]$, then $x = \tilde{A}(u)^{-p} \in (0, 1]$ and $y = \tilde{B}(u)^{-p} \in (0, 1]$. We know $x + y > 1$, then $0 < x + y - 1 < 1$.

Because $y \leq 1$ then $y - 1 \leq 0$, that is $0 < x + y - 1 < x$.

This means $0 < \frac{x+y-1}{x} \leq 1$, that is $0 < x^{1-\frac{1}{x}+\frac{y}{x}} \leq 1$.

Similarly we have $0 < \frac{x+y-1}{y} \leq 1$, that is $0 < y^{1-\frac{1}{y}+\frac{x}{y}} \leq 1$.

This means $G(p) \leq 0$. Because $F(p) > 0$, we can introduce

$$\frac{\partial}{\partial p} F(p) = F(p) \bullet \frac{\partial}{\partial p} \left\{ \frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p} \right\} = F(p) \bullet G(p) \leq 0$$

This means that when $p < 0$, formula (1.1) is monotone decreasing function for the parameter p .

Theorem 2.3 When $p > 0$, formula (1.2) is monotone decreasing function for the parameter p .

Proof. Omitted.

This means that when $p > 0$, formula (1.2) is monotone decreasing function for the parameter p .

Theorem 2.4 When $p < 0$, formula (1.2) is monotone increasing function for the parameter p .

Proof. Omitted.

This means that when $p < 0$, formula (1.2) is monotone increasing function for the parameter p .

The formula (1.1) and the formula (1.2) have defined infinite fuzzy operators. It is quite evident that from the theorem 2.1 to the theorem 2.4 have given a sort of continued Fuzzy operators^[7,8]. This is a new important property of Schweizer-Sklard operators.

3. THE RELATION OF THE SCHWEIZER-SKLARD OPERATIONS AND OTHER FUZZY OPERATIONS MOST IN USE

To any fuzzy set $\tilde{A}, \tilde{B} \in P(U)$, $\forall u \in U$, Zadeh operator and three other common generalized operators (probability operators, boundary operators and infinite operators) have relation^[9,10]:

$$\tilde{A}(u) \circ \tilde{B}(u) \subseteq \tilde{A}(u) \otimes \tilde{B}(u) \subseteq \tilde{A}(u) \hat{\bullet} \tilde{B}(u) \subseteq \tilde{A}(u) \wedge \tilde{B}(u)$$

$$\subseteq \tilde{A}(u) \vee \tilde{B}(u) \subseteq \tilde{A}(u) \hat{+} \tilde{B}(u) \subseteq \tilde{A}(u) \oplus \tilde{B}(u) \subseteq \tilde{A}(u) \overset{+}{\sim} \tilde{B}(u)$$

For the Schweizer-Sklard operators $(\overset{\bullet}{s}s, \overset{+}{s}s)$, we give a new conclusion as follows:

Theorem 3.1 $\forall \tilde{A}, \tilde{B} \in P(U), \forall u \in U$, For the parameter $p \rightarrow +\infty$, the continued Fuzzy operators, that is, Schweizer-Sklard operators $(\overset{\bullet}{s}s, \overset{+}{s}s)$ and Zadeh operators (\wedge, \vee) have relation as follows:

$$[\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)]_{+\infty} = \tilde{A}(u) \wedge \tilde{B}(u) \subseteq \tilde{A}(u) \vee \tilde{B}(u) = [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)]_{+\infty} \quad (3.1)$$

Proof. If the parameter $p \rightarrow +\infty$, then $\forall u \in U$, the $\tilde{C}(u)$ in definition 2.1 is denoted $[\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)]_{+\infty}$ and $[\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)]_{+\infty}$ as follows:

$$\lim_{p \rightarrow +\infty} [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)] = [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)]_{+\infty}, \lim_{p \rightarrow +\infty} [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)] = [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)]_{+\infty}.$$

Zadeh operators (\wedge, \vee) are denoted by

$$\tilde{A}(u) \wedge \tilde{B}(u) = \min[\tilde{A}(u), \tilde{B}(u)], \tilde{A}(u) \vee \tilde{B}(u) = \max[\tilde{A}(u), \tilde{B}(u)].$$

We subdivide proof into two steps:

(a) $\forall \tilde{A}, \tilde{B} \in P(U), \forall u \in U$, when the parameter $p > 0$, if $\tilde{A}(u)\tilde{B}(u)=0$, then

$$\lim_{p \rightarrow +\infty} [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)] = 0 = \min[\tilde{A}(u), \tilde{B}(u)] = \tilde{A}(u) \wedge \tilde{B}(u).$$

If $\tilde{A}(u)\tilde{B}(u) \neq 0$, let $\tilde{A}(u) \geq \tilde{B}(u)$ then

$$\begin{aligned} \lim_{p \rightarrow +\infty} [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)] &= \lim_{p \rightarrow +\infty} [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} \\ &= \lim_{p \rightarrow +\infty} \exp \frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p} \\ &= \lim_{p \rightarrow +\infty} \exp \frac{\tilde{A}(u)^{-p} \ln \tilde{A}(u) + \tilde{B}(u)^{-p} \ln \tilde{B}(u)}{\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1} \\ &= \tilde{B}(u) = \tilde{A}(u) \wedge \tilde{B}(u) \end{aligned}$$

(b) $\forall \tilde{A}, \tilde{B} \in P(U), \forall u \in U$, when the parameter $p > 0$, if $\tilde{A}(u)=1$ or $\tilde{B}(u)=1$, then

$$\lim_{p \rightarrow +\infty} [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)] = 1 = \max[\tilde{A}(u), \tilde{B}(u)] = \tilde{A}(u) \vee \tilde{B}(u)$$

If $\tilde{A}(u)\tilde{B}(u) \neq 1$, let $\tilde{A}(u) \geq \tilde{B}(u)$ then

$$\begin{aligned} \lim_{p \rightarrow +\infty} [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)] &= \lim_{p \rightarrow +\infty} \{1 - \{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}^{-1/p}\} \\ &= 1 - \lim_{p \rightarrow +\infty} \exp \frac{\ln\{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}}{-p} \\ &= 1 - \lim_{p \rightarrow +\infty} \exp \frac{[1 - \tilde{A}(u)]^{-p} \ln[1 - \tilde{A}(u)] + [1 - \tilde{B}(u)]^{-p} \ln[1 - \tilde{B}(u)]}{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1} \\ &= \tilde{A}(u) = \tilde{A}(u) \vee \tilde{B}(u) \end{aligned}$$

It is quite evident that (a) and (b) implies (3.1).

Theorem 3.2 $\forall \tilde{A}, \tilde{B} \in P(U), \forall u \in U$, for the parameter $p \rightarrow +0$, the continued Fuzzy operators, that is, Schweizer-Sklard operators $(\overset{\bullet}{s}s, \overset{+}{s}s)$ and probability operators $(\hat{\bullet}, \hat{+})$ have relation as follows:

$$\tilde{A}(u) \hat{\bullet} \tilde{B}(u) = [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)]_{+0} \subseteq [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)]_{+0} = \tilde{A}(u) \hat{+} \tilde{B}(u) \quad (3.2)$$

Proof. If the parameter $p \rightarrow +0$, then $\forall u \in U$, the $\tilde{C}(u)$ in definition 1.1 is denoted $[\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)]_{+0}$ and $[\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)]_{+0}$ as follows:

$$\lim_{p \rightarrow +0} [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)] = [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)]_{+0}, \lim_{p \rightarrow +0} [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)] = [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)]_{+0}.$$

Zadeh operators ($\overset{\bullet}{\wedge}, \overset{\hat{+}}{\wedge}$) are denoted by

$$\tilde{A}(u) \overset{\bullet}{\wedge} \tilde{B}(u) = \tilde{A}(u) \tilde{B}(u), \tilde{A}(u) \overset{\hat{+}}{\wedge} \tilde{B}(u) = \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u) \tilde{B}(u).$$

We subdivide proof into two steps:

(a) $\forall \tilde{A}, \tilde{B} \in P(U), \forall u \in U$, when the parameter $p > 0$, if $\tilde{A}(u) \tilde{B}(u) = 0$, then

$$\lim_{p \rightarrow +\infty} [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)] = 0 = \tilde{A}(u) \tilde{B}(u) = \tilde{A}(u) \overset{\bullet}{\wedge} \tilde{B}(u)$$

If $\tilde{A}(u) \tilde{B}(u) \neq 0$, let $\tilde{A}(u) \geq \tilde{B}(u)$ then

$$\begin{aligned} \lim_{p \rightarrow +0} [\tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u)] &= \lim_{p \rightarrow +0} [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} \\ &= \lim_{p \rightarrow +0} \exp \frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p} \\ &= \lim_{p \rightarrow +0} \exp \frac{\tilde{A}(u)^{-p} \ln \tilde{A}(u) + \tilde{B}(u)^{-p} \ln \tilde{B}(u)}{\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1} = \tilde{A}(u) \tilde{B}(u) = \tilde{A}(u) \overset{\bullet}{\wedge} \tilde{B}(u) \end{aligned}$$

(b) $\forall \tilde{A}, \tilde{B} \in P(U), \forall u \in U$, when the parameter $p > 0$, if $\tilde{A}(u) = 1$ or $\tilde{B}(u) = 1$, let $\tilde{A}(u) = 1$ then

$$\lim_{p \rightarrow +0} [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)] = 1 = 1 + \tilde{B}(u) - 1 \bullet \tilde{B}(u) = \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u) \tilde{B}(u) = \tilde{A}(u) \overset{\hat{+}}{\wedge} \tilde{B}(u)$$

If $\tilde{A}(u) \tilde{B}(u) \neq 1$, let $\tilde{A}(u) \geq \tilde{B}(u)$ then

$$\begin{aligned} \lim_{p \rightarrow +0} [\tilde{A}(u) \overset{+}{s}s \tilde{B}(u)] &= \lim_{p \rightarrow +0} \{1 - \{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}^{-1/p}\} \\ &= 1 - \lim_{p \rightarrow +0} \exp \frac{\ln\{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}}{-p} \\ &= 1 - \lim_{p \rightarrow +0} \exp \frac{[1 - \tilde{A}(u)]^{-p} \ln[1 - \tilde{A}(u)] + [1 - \tilde{B}(u)]^{-p} \ln[1 - \tilde{B}(u)]}{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1} \\ &= \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u) \tilde{B}(u) \\ &= \tilde{A}(u) \overset{\hat{+}}{\wedge} \tilde{B}(u) \end{aligned}$$

It is quite evident that (a) and (b) implies (3.2).

From theorem 2.1 to theorem 3.2 implies that continued Fuzzy operators, Schweizer-Sklard operations, could replace other fuzzy operations most in use. At the same time, when the parameter $p > 0$, from theorem 2.1 to theorem 3.2 implies that continued Fuzzy operators, Schweizer-Sklard operations, and Zadeh operations as well as probability operators have relation as follows:

$$\tilde{A}(u) \overset{\bullet}{\wedge} \tilde{B}(u) \subseteq \tilde{A}(u) \overset{\bullet}{s}s \tilde{B}(u) \subseteq \tilde{A}(u) \wedge \tilde{B}(u) \subseteq \tilde{A}(u) \vee \tilde{B}(u) \subseteq \tilde{A}(u) \overset{+}{s}s \tilde{B}(u) \subseteq \tilde{A}(u) \overset{\hat{+}}{\wedge} \tilde{B}(u) \quad (3.3)$$

CONCLUSION

From theorem 2.1 to theorem 3.2 implies that Schweizer-Sklard operations are continued Fuzzy operators and that Schweizer-Sklard operations could partly replace other fuzzy operations most in use.

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